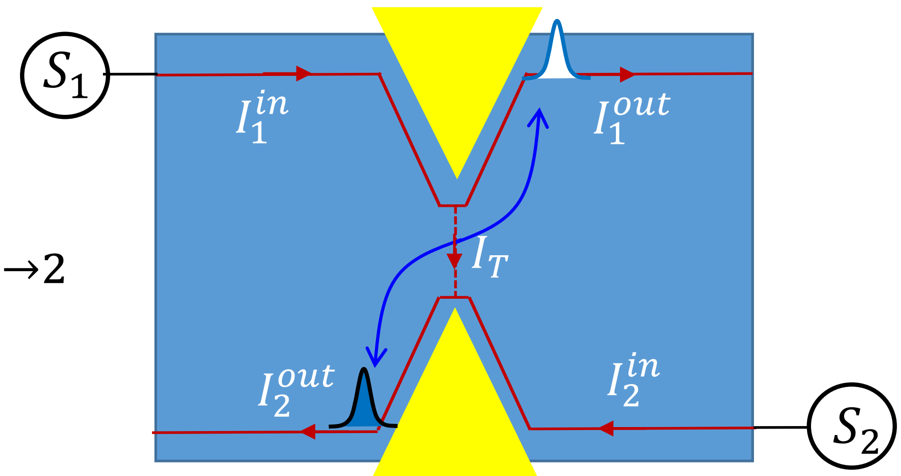
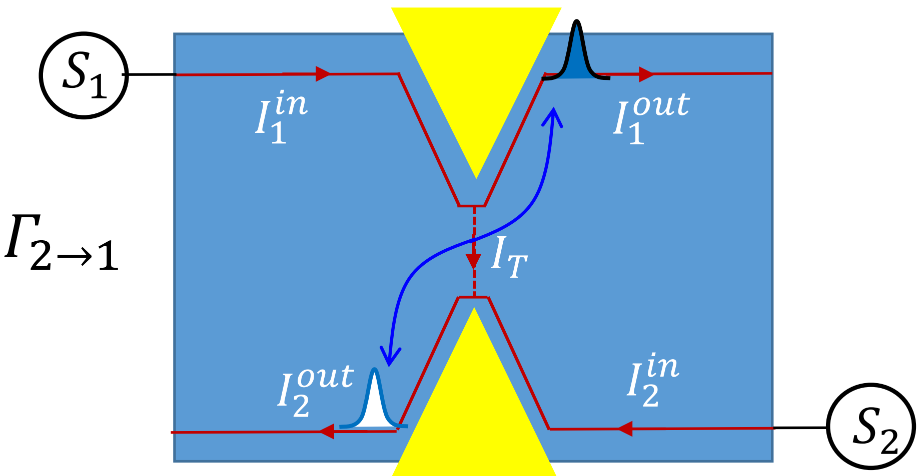
Noise measurements for the characterization of anyons in the fractional quantum Hall effect (2)

Weak backscattering regime: lowest order in tunneling $H_T = \zeta \psi^{+}_{1,\alpha}\psi_{2,\alpha} + \zeta^{*}\psi^{+}_{2,\alpha}\psi_{1,\alpha}$

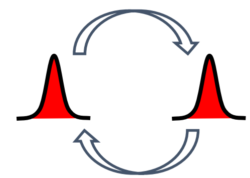
$\Gamma_{1\to 2}$



$\Gamma_{2\to 1}$

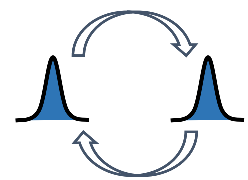


$\nu =$ integer: $\psi^{+}_{1,\alpha} = \psi^{+}_{1,e}$ particles are electrons, $e^{*} = e$



$\varphi = \pi,\ \theta = 2\varphi = 2\pi$

$\nu = 1/3$: $\psi^{+}_{1,\alpha} = \psi^{+}_{1,a}$ particles are anyons, $e^{*} = e/3$



$\varphi = \pi/3, \quad \theta = 2\pi/3$

$$I_T = e^{*}(\Gamma_{2\to 1} - \Gamma_{1\to 2})$$

$$S_{I_T} = 2e^{*2}(\Gamma_{1\to 2} + \Gamma_{2\to 1})$$

$$S_{12}^{out} = -S_{I_T} + T\big[\underbrace{S_{11}^{in} + S_{22}^{in}}_{S_{\Sigma}^{in}\text{: input noise}}\big]$$

$$\Gamma_{1\to 2} \propto \int d\tau \left\langle \psi_{1,a}^{+,in}(\tau)\psi_1^{in}(0)\right\rangle \left\langle \psi_{2,a}^{in}(\tau)\psi_2^{+,in}(0)\right\rangle$$

$$\Gamma_{2\to 1} \propto \int d\tau \left\langle \psi_{2,a}^{+,in}(\tau)\psi_{2,a}^{in}(0)\right\rangle \left\langle \psi_{1,a}^{in}(\tau)\psi_{1,a}^{+,in}(0)\right\rangle$$

Example: voltage biased QPC

Fano factor: $F = \frac{S_{I_T}}{2e^* I_T} = \frac{\Gamma_{1\to 2} + \Gamma_{2\to 1}}{\Gamma_{2\to 1} - \Gamma_{1\to 2}}$

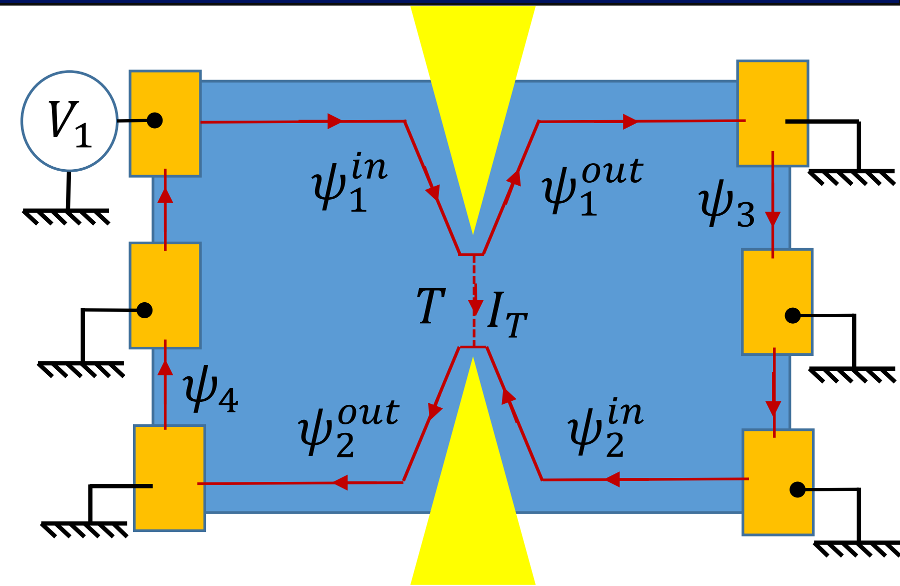
$\Gamma_{2\to 1} = T\frac{eV}{h} = T\Gamma_0$

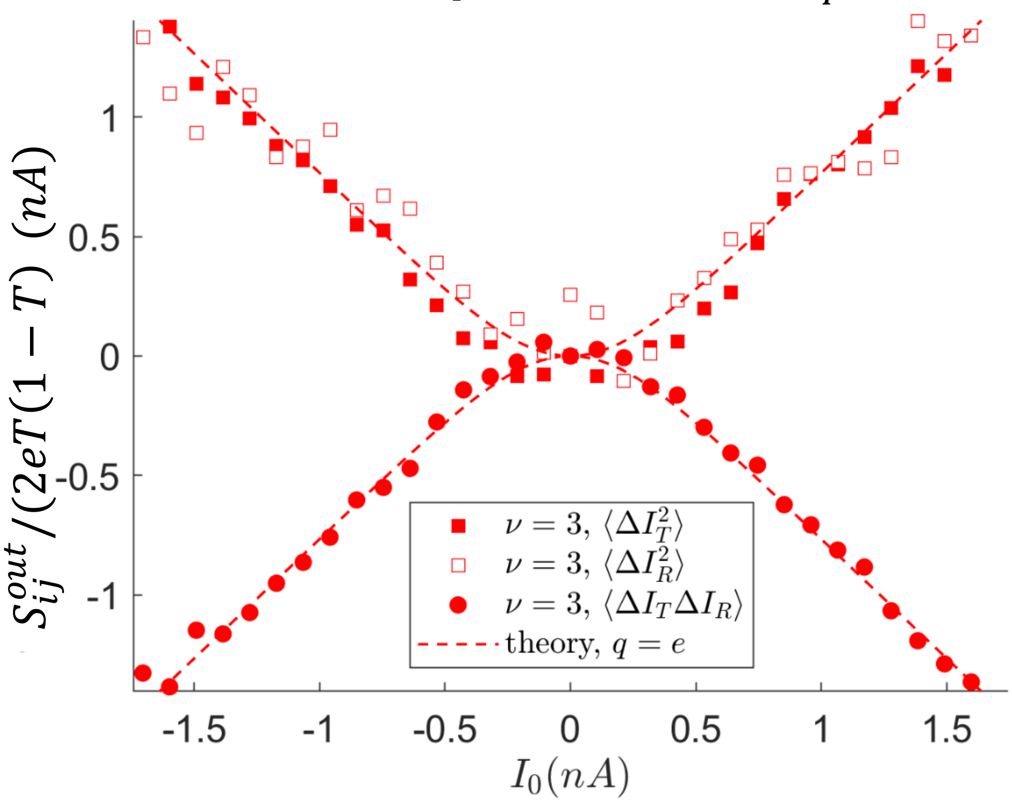
$\Gamma_{1\to 2} = 0$

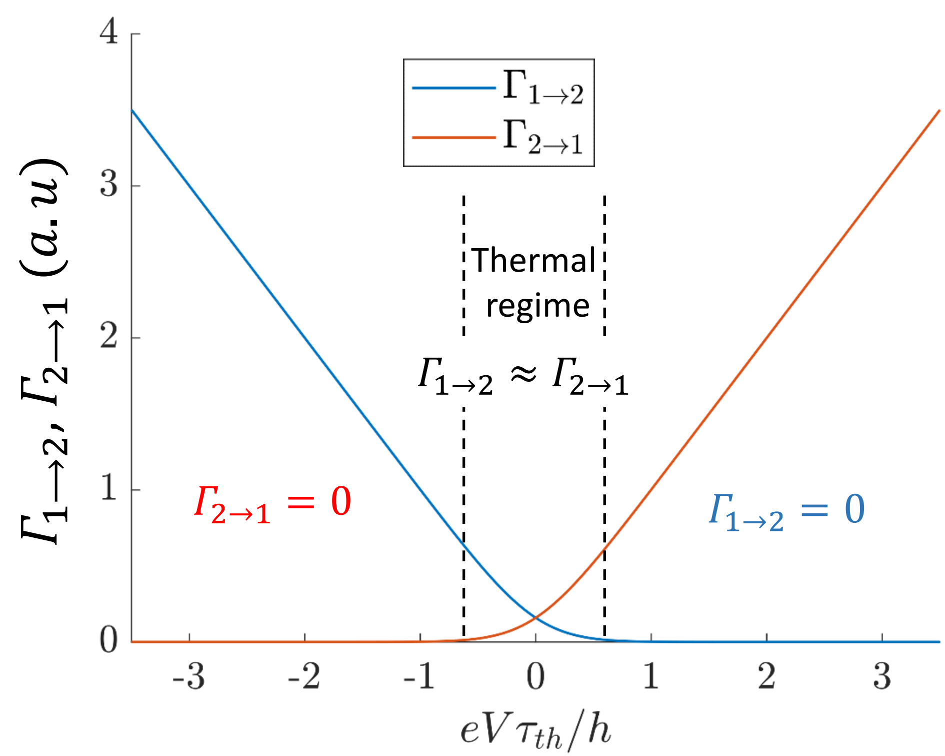
$\Rightarrow F = 1 \Rightarrow S_{I_T} = 2eI_T$

No noise at the input: $S_{11}^{in} = 0$

$S_{12}^{out} = -S_{I_T}$ $S_{11}^{out} = +S_{I_T}$







Fractional case, equilibrium correlation function

$$\left\langle GS|\psi^{+,in}_{1,a}(\tau)\psi^{in}_{1,a}(0)|GS\right\rangle = G_{eq,\delta}(\tau) = \frac{1}{2\pi\tau_c}\left[\frac{\sinh\left[i\frac{\pi\tau_c}{\tau_{th}}\right]}{\sinh\left[i\frac{\pi\tau_c}{\tau_{th}}-\frac{\pi\tau}{\tau_{th}}\right]}\right]^{\delta} \qquad \tau_{th} = \frac{\hbar}{k_B T_{el}}$$

X.G. Wen, Advances in Physics, **44**, 405 (1995)

T. Martin, les Houches Session LXXXI, arXiv:cond-mat/0501208

$G_{eq,\delta=1}(\tau)$ Fourier transform of the Fermi sea, correlations of the fermi liquid

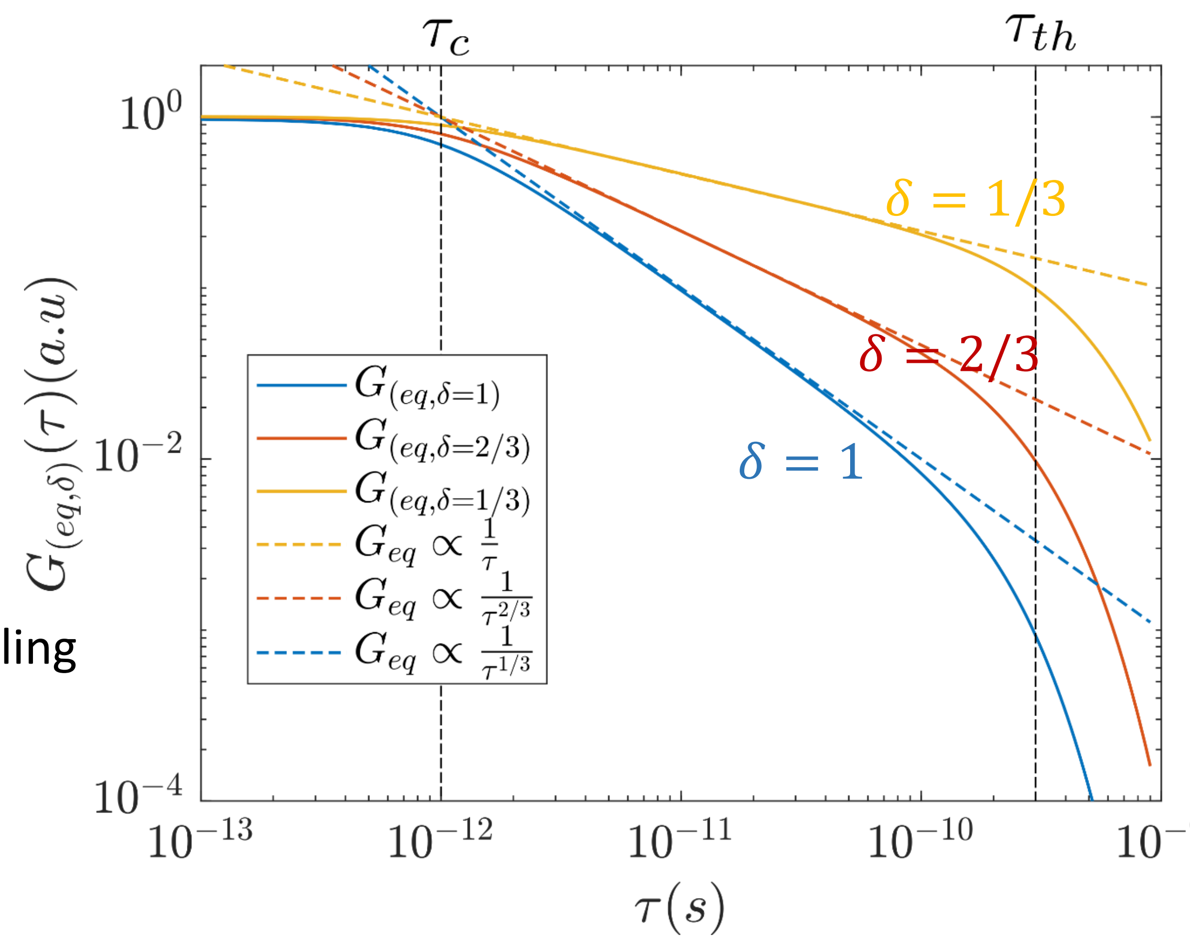
$\delta < 1$, long time correlations:

$$\tau \ll \tau_{th}, G_{eq,\delta}(\tau) \approx \left[\frac{1}{i\pi(t-t'-i\tau_c)}\right]^{\delta}$$

Laughlin state: $\nu = \delta = 1/m$

δ : scaling dimension for anyon tunnneling

$$G_{eq,\delta}(-\tau) = \left[G_{eq,\delta}(\tau)\right]^*$$



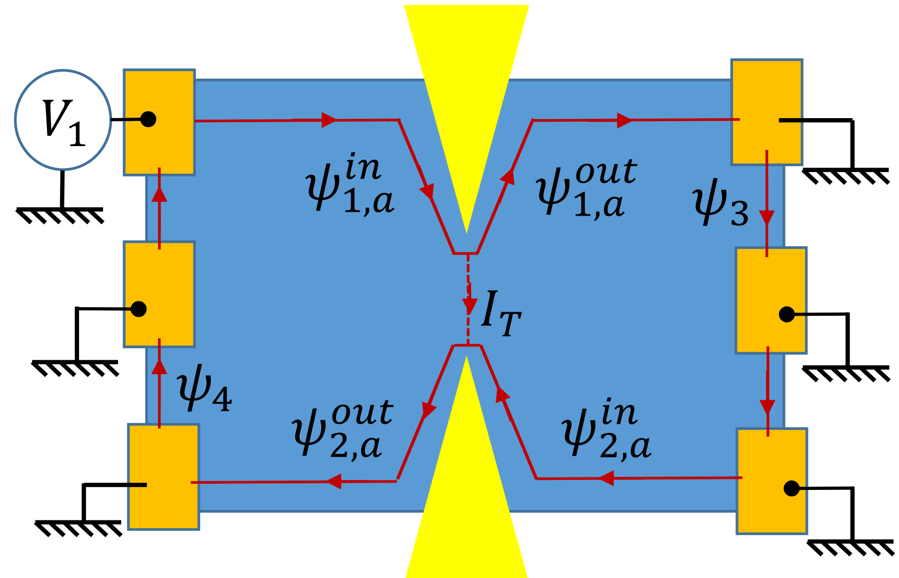
Single quantum point contact in the dc regime: random poissonian emission of anyons

LPENS
LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

$$\left\langle \psi_{1,a}^{+,in}(\tau)\psi_{1,a}^{in}(0)\right\rangle = e^{-i\frac{e^* V_1 \tau}{\hbar}} G_{eq,\delta}(\tau)$$

$$\left\langle \psi_{1,a}^{in}(\tau)\psi_{1,a}^{+,in}(0)\right\rangle = e^{i\frac{e^* V_1 \tau}{\hbar}} G_{eq,\delta}(\tau)$$

$$\Gamma_{1\to 2} \propto 2Re\left[\int_0^{+\infty} d\tau\, e^{-i\frac{e^* V_1 \tau}{\hbar}} \left[G_{eq,\delta}(\tau)\right]^2\right]$$



$$I(z) = \int_0^{+\infty} du \left[\frac{\sinh[iu_c]}{\sinh[iu_c - u]}\right]^{2\delta} e^{2izu} \propto e^{-i\pi\delta} \frac{\Gamma(\delta - iz)}{\Gamma(1-\delta-iz)} \approx e^{-i\pi\delta}(-iz)^{2\delta-1} \text{ for } z \gg 1$$

$$\Gamma_{1\to 2} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta+iz)}{\Gamma(1-\delta+iz)}\right] \text{ with } z = \frac{e^* V \tau_{th}}{h} \approx 2Re\left[e^{-i\pi\delta}(iz)^{2\delta-1}\right] = 2z^{2\delta-1} Re\left[e^{-i\pi\delta}e^{+i\pi\delta}(-i)\right] = 0$$

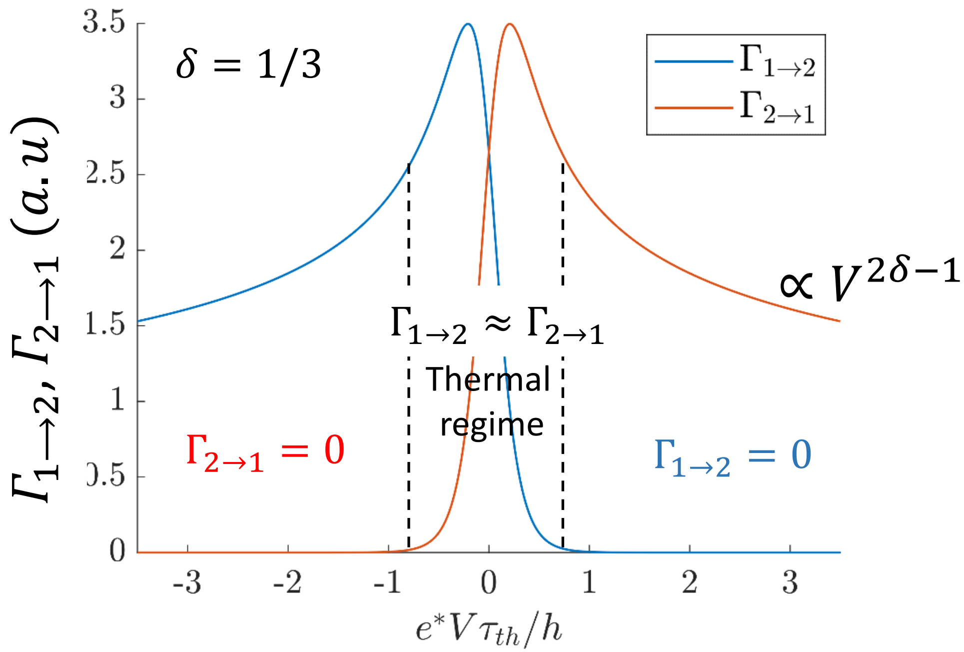
$$\Gamma_{2\to 1} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta-iz)}{\Gamma(1-\delta-iz)}\right] \text{ with } z = \frac{e^* V \tau_{th}}{h} \approx 2Re\left[e^{-i\pi\delta}(-iz)^{2\delta-1}\right] = 2z^{2\delta-1} Re\left[e^{-i\pi\delta}e^{-i\pi\delta} i\right] \neq 0$$

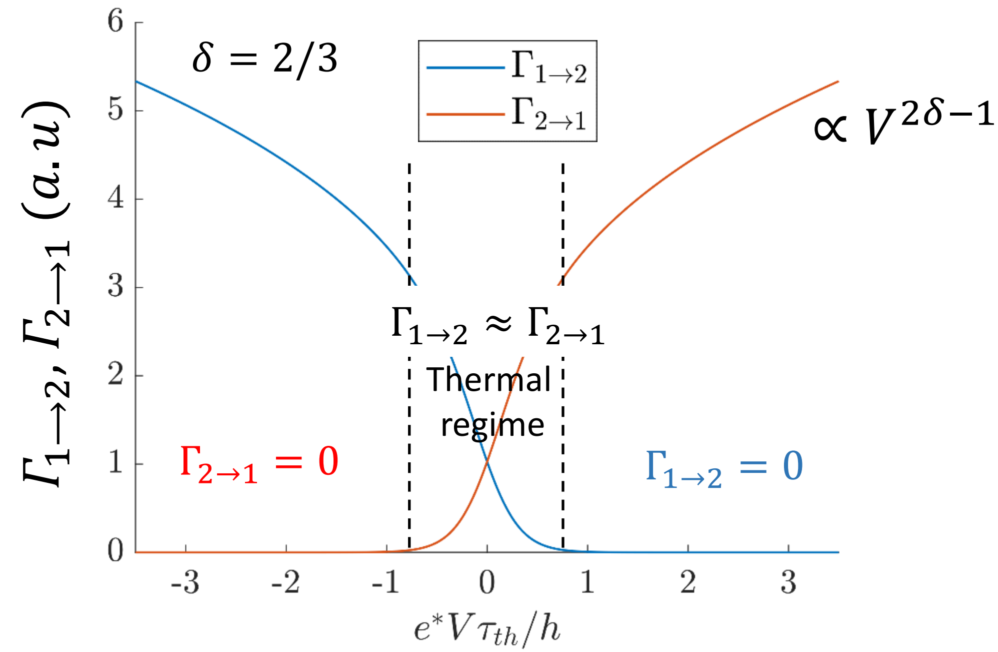
$$\left.\begin{array}{lll} V_1 > 0 & \Gamma_{1\to 2} = 0 & \Gamma_{2\to 1} \neq 0 \\ V_1 < 0 & \Gamma_{1\to 2} \neq 0 & \Gamma_{2\to 1} = 0 \end{array}\right\}$$

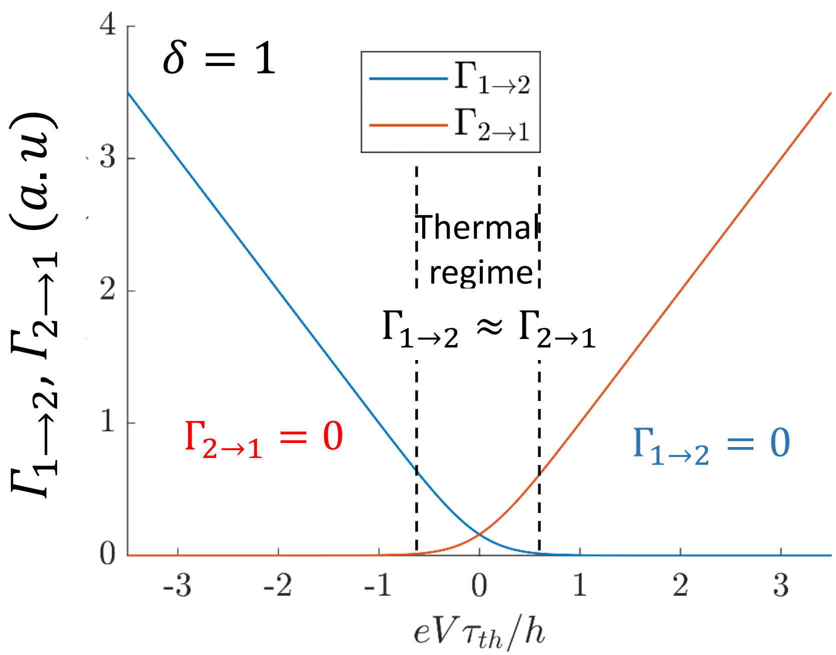
$$\boxed{F = \frac{S_{I_T}}{2e^* I_T} = 1 \Rightarrow S_{I_T} = 2e^* I_T}$$

C.L. Cane, M.P.A. Fisher, PRL, **72**, 724 (1994)

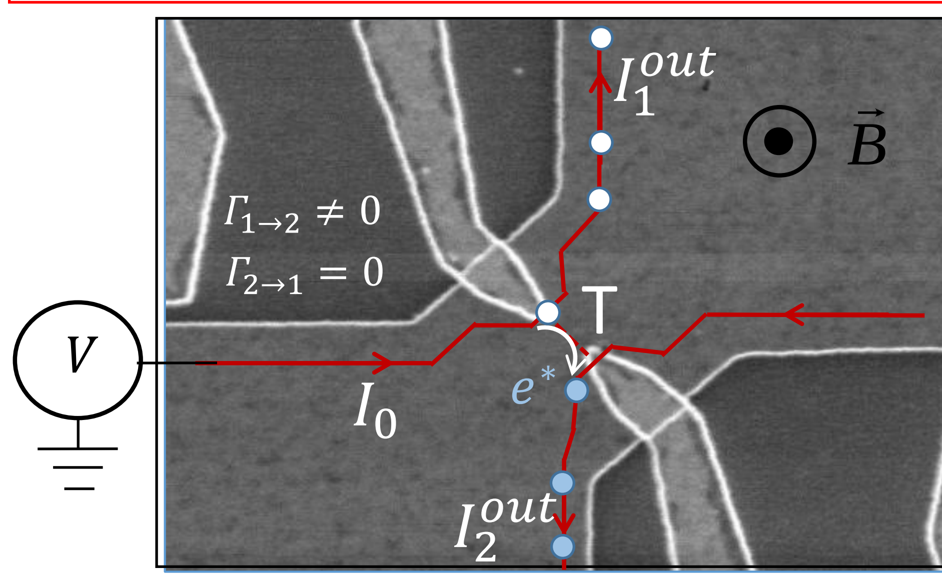
Single quantum point contact in the dc regime: random poissonian emission of anyons



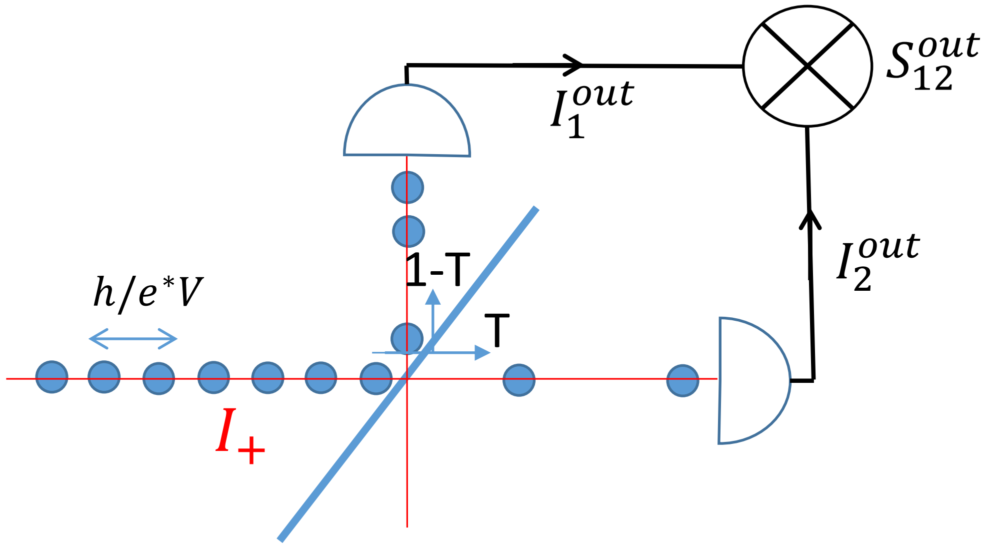




$\boxed{\left|\frac{e^* V \tau_{th}}{h}\right| \gg 1\text{: random poissonian emission of anyons}}$

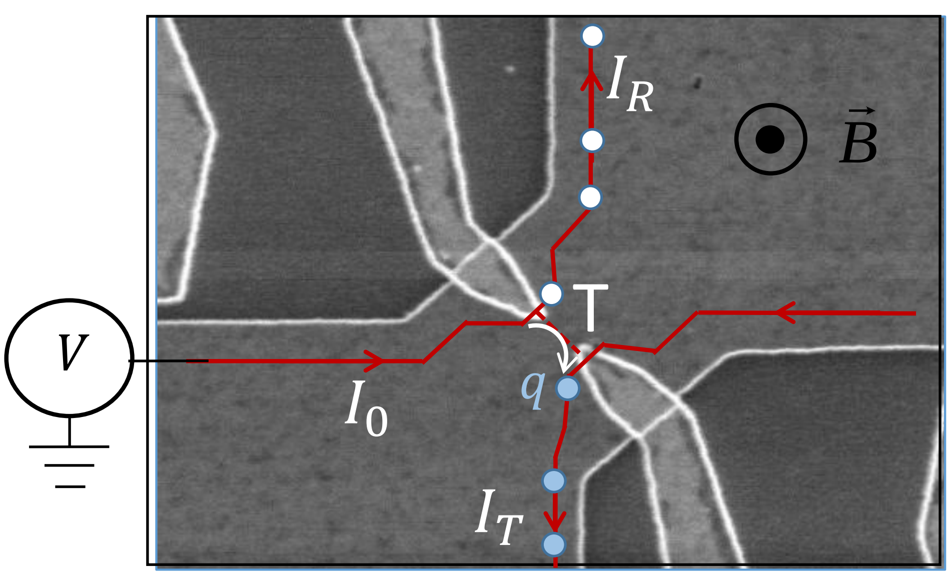


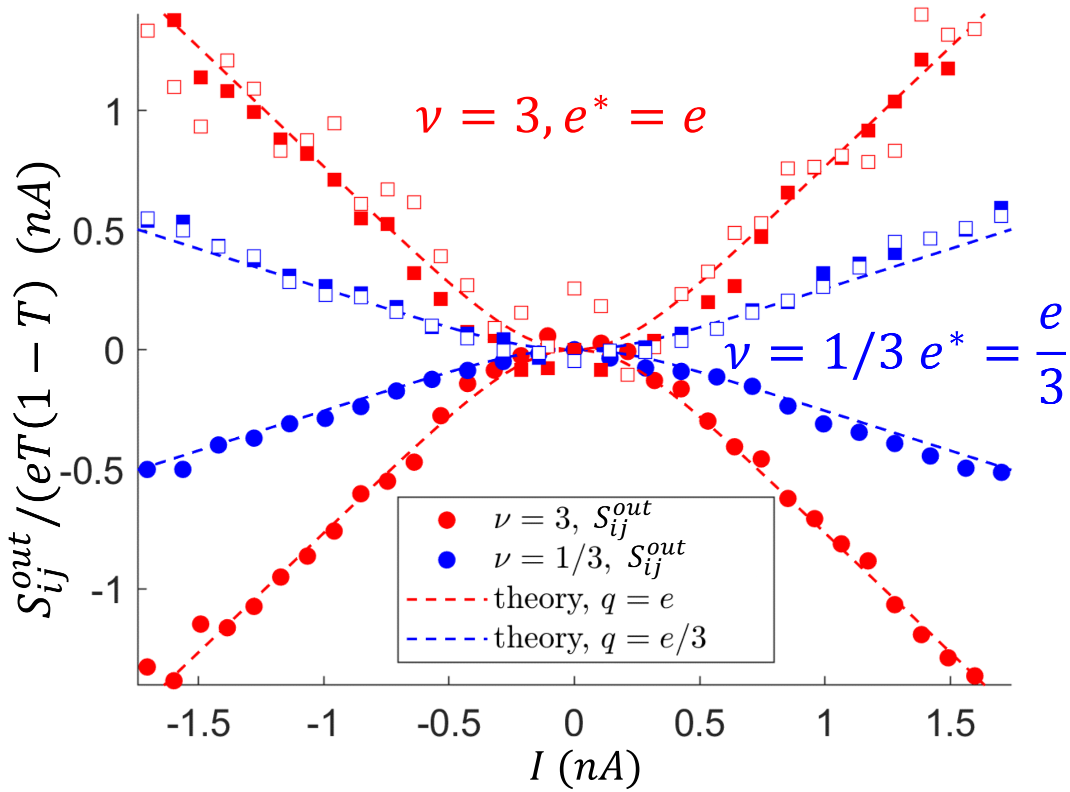
Single quantum point contact in the dc regime: anyon fractional charge



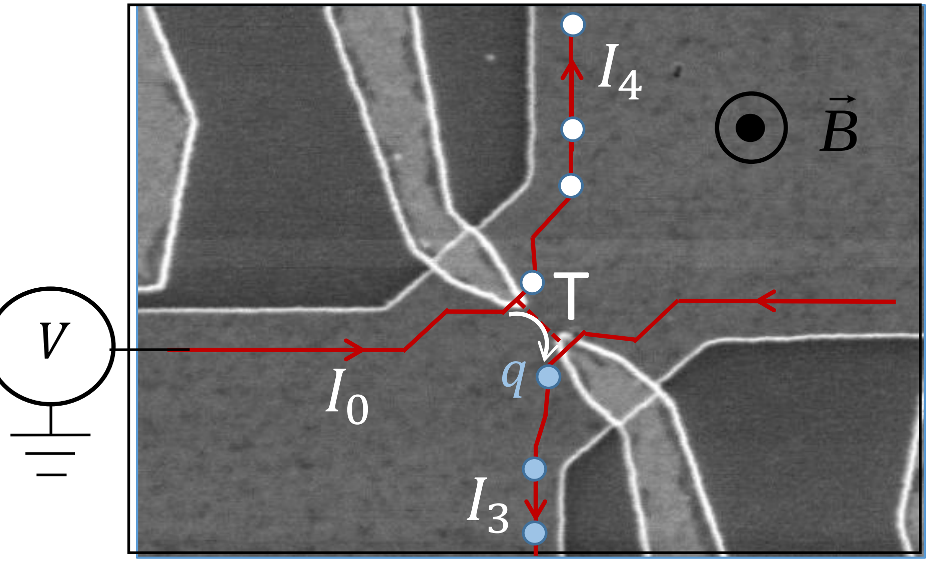
$$\boxed{S_{I_T} = 2e^* I_T}$$

$$S_{12}^{out} = -S_{11}^{out} = -S_{22}^{out} = S_{rp}$$





Single quantum point contact in the dc regime: anyon fractional charge

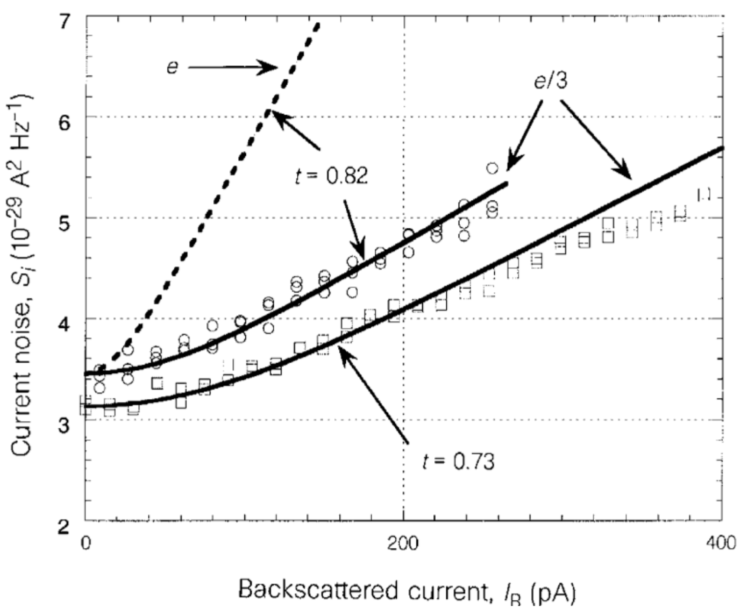


$\left|\frac{e^* V \tau_{th}}{h}\right| \gg 1$: random poissonian emission of anyons

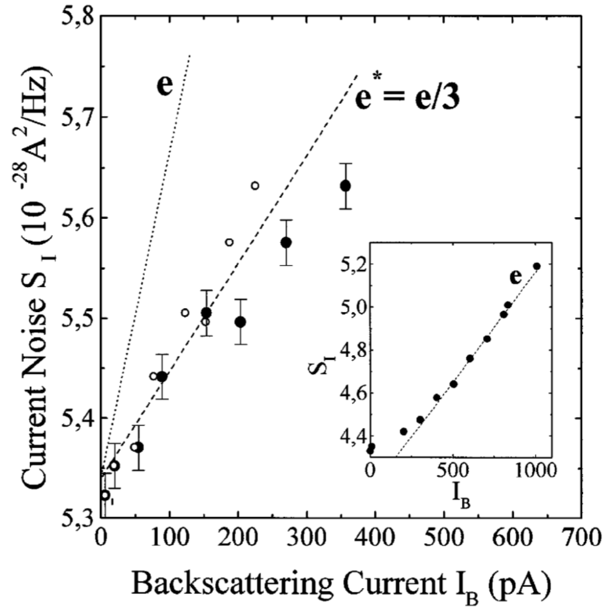
$$\boxed{S_{I_T} = 2e^* I_T}$$

$$S_{12}^{out} = -S_{11}^{out} = -S_{22}^{out} = S_{rp}$$

Fractional case: $e^* = e/3$

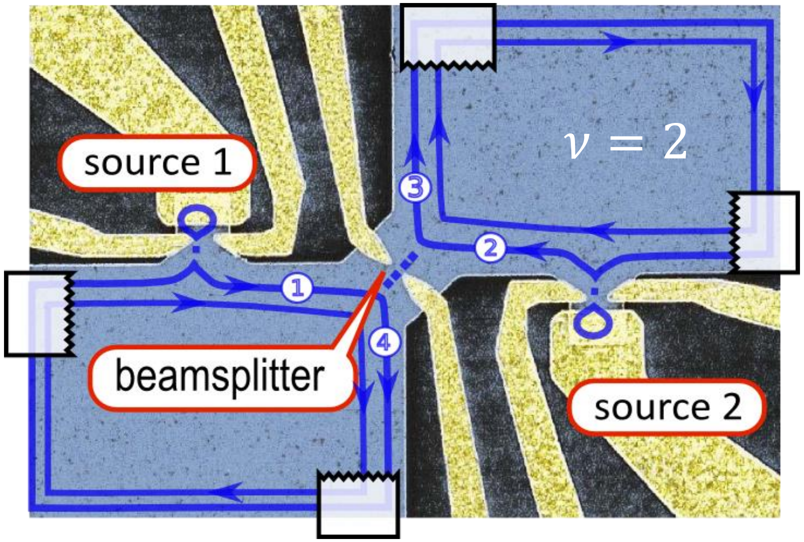


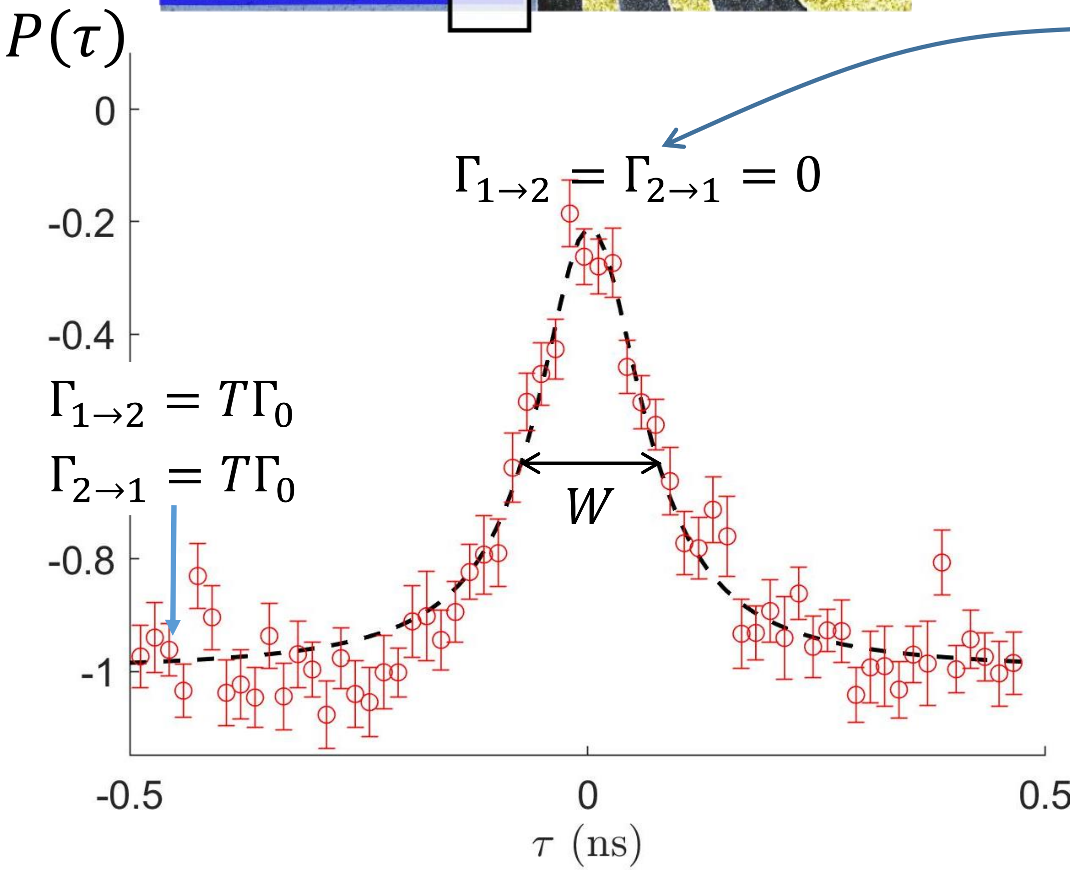
R. de Picciotto et al., Nature **389**, 162 (1997).

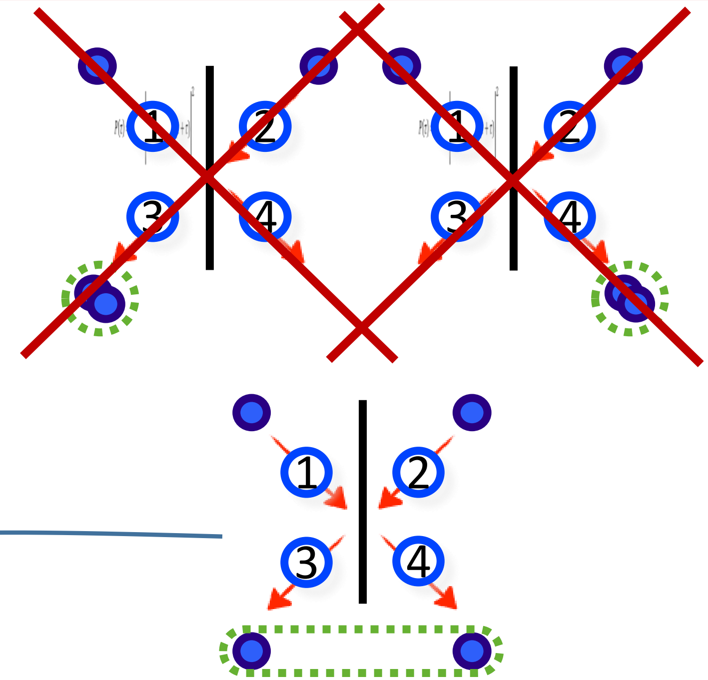


L. Saminadayar et al., Phys. Rev. Lett. **79**, 2526 (1997).

Hong-Ou-Mandel experiment with electrons (integer quantum Hall effect)







$$I_1^{in} = I_2^{in} \longrightarrow \Gamma_{1\to2} = \Gamma_{2\to1} \longrightarrow I_T = 0$$

- $\tau \gg W$ Classical random partitioning

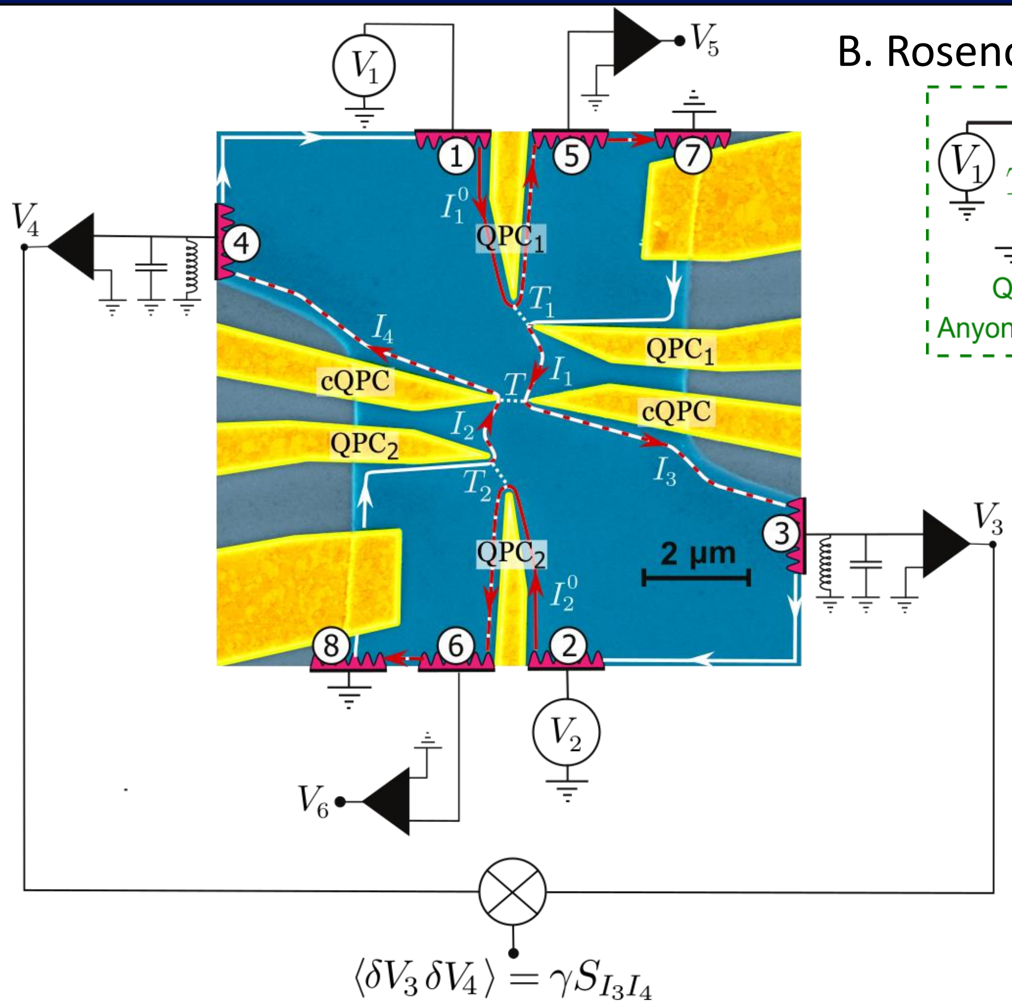
$$P(\tau) = \frac{S_{12}^{out}}{2eTI_+} = -1 \qquad I_+ = I_1 + I_2$$

- $\tau \ll W$ Fermion antibunching

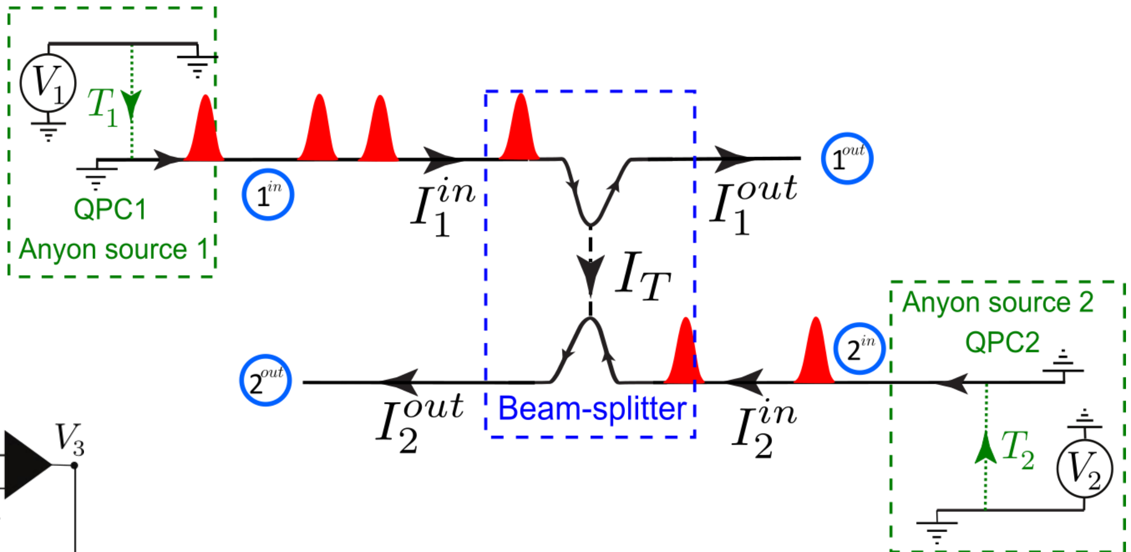
$$P(\tau) \approx 0$$

S. Ol'khovskaya et al., PRL **101**, 166802, (2008).
E. Bocquillon et al., Science **339**, 1054 (2013).

The anyon collider



B. Rosenow, I.P. Levkivskyi, B. Halperin PRL **116**, 156802 (2016)



Random emission of particles: probabilities $T_1 = T_2 = T_S$

Balanced case: $V_1 = V_2 \neq 0,$: $I_1^{in} = I_2^{in}$

Unbalanced case: $V_1 \neq 0, V_2 = 0$: $I_1^{in} \neq 0, I_2^{in} = 0$

Total input current: $I_+ = I_1^{in} + I_2^{in}$

Current difference: $I_- = I_1^{in} - I_2^{in}$

Tunneling current: I_T

Beam-splitter of transmission $T = \frac{\partial I_T}{\partial I_-}$

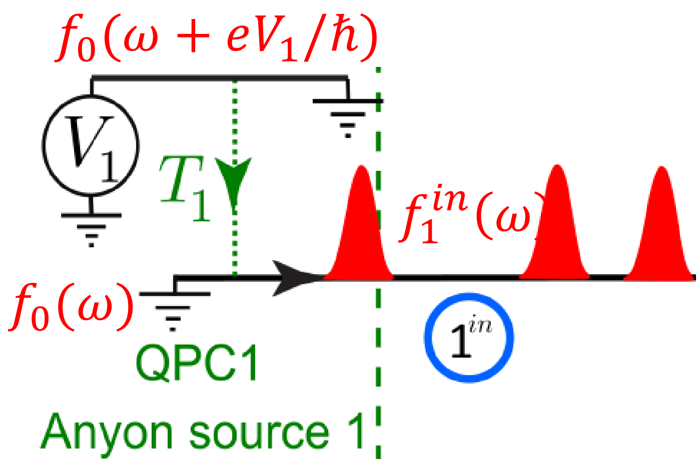
Fano factors:

$I_1^{in} = I_2^{in}$ $\quad P = \frac{S_{12}^{out}}{2e^* T I_+}$

$I_1^{in} \neq 0, I_2^{in} = 0$ $\quad P = \frac{S_{12}^{out}}{2e^* T I_+}$ $\quad F = \frac{S_{I_T}}{2e^* I_T}$

The fermion case, exact calculation from the rates
Unbalanced case $I_1^{in} \neq 0, I_2^{in} = 0$

Unbalanced collider: $V_2 = 0, V_1 < 0$

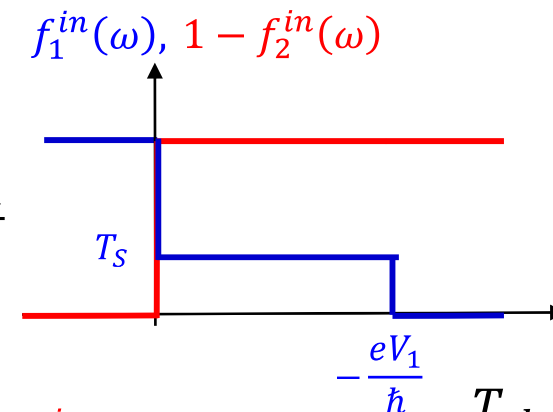


$$f_1^{in}(\omega) = (1 - T_S) f_0(\omega) + T_S f_0(\omega + eV_1/\hbar)$$

$$f_2^{in}(\omega) = f_0(\omega)$$

$$\boxed{F_F = 1}$$

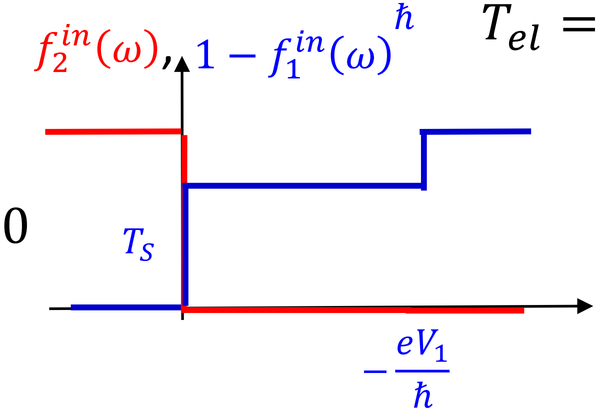
$$\Gamma_{1\to 2} = T T_S \frac{-eV_1}{h}$$



$$T_{el} = 0$$

$$S_{12}^{out} = \underbrace{2e^2 T I_1^{in}(1 - T_S)}_{T S_{11}^{in}} - \underbrace{2e^2 T I_1^{in}}_{-S_{I_T}} = -T_S 2e^2 T I_1^{in}$$

$$\Gamma_{2\to 1} = 0$$

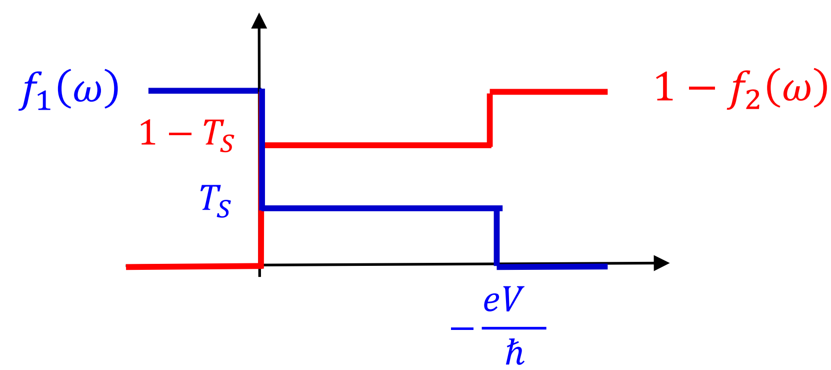


$$\boxed{P_F = \frac{S_{12}^{out}}{2eTI_1^{in}} = -T_S}$$

Poissonian limit: $T_S \ll 1, \quad P_F = 0$

The fermion case, exact calculation from the rates

Balanced case: $T_1 = T_2 = T_S < 0 \qquad V_1 = V_2 = V < 0$



$$\Gamma_{1\to 2} = T T_S (1 - T_S) \frac{-eV}{h}$$

$$\Gamma_{2\to 1} = T T_S (1 - T_S) \frac{-eV}{h}$$

$$S_{I_T} = 4e^2 T T_S (1 - T_S) \frac{-eV}{h} = 2eTI_+ - 4e^2 T T_S^2 \frac{-eV}{h} = 2e^2 T I_+ (1 - T_S)$$

S_{cl} fermion antibunching

$$S_{12}^{out} = \underbrace{2e^2 T I_+ (1 - T_S)}_{T\left[S_{11}^{in} + S_{22}^{in}\right]} - \underbrace{2e^2 T I_+ (1 - T_S)}_{-S_{I_T}} = 0$$

$$\boxed{P_F\left(I_1^{in} = I_2^{in}\right) = 0}$$

(fermion antibunching)

Fermions and bosons: classical in the Poissonian limit

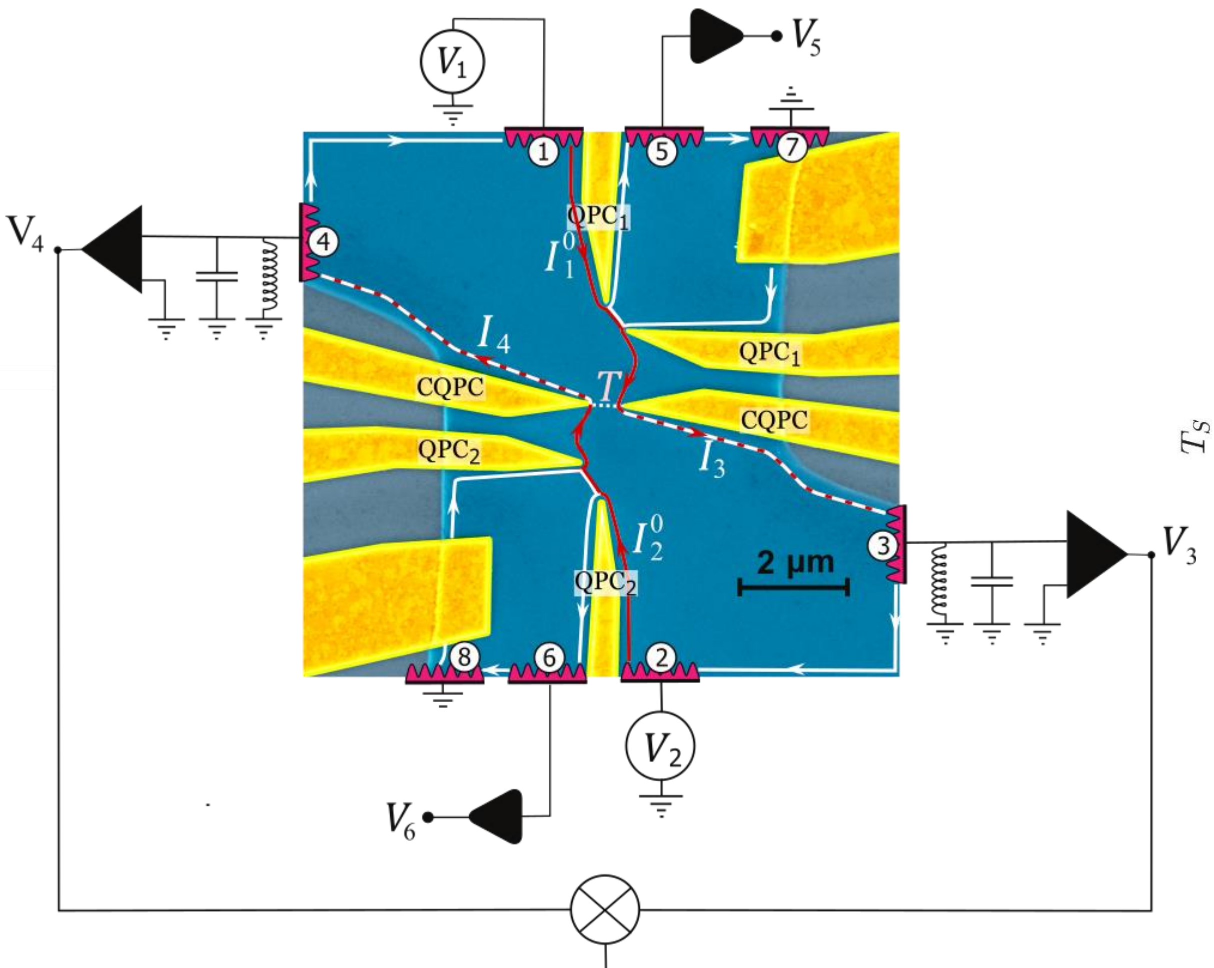
Poissonian limit: $T_S \ll 1$,	Fermions	Bosons
Unbalanced collider: $I_1^{in} \neq 0, I_2^{in} = 0$	$F_F = 1 \quad P_F = 0$	$F_B = 1 \quad P_B = 0$
Balanced collider: $I_1^{in} = I_2^{in}$	$P_F = 0$	$P_B = 0$

Fermion antibunching and boson bunching are suppressed in the Poissonian limit:
Probability to have two particles in the interferometer $\propto T_S^2$

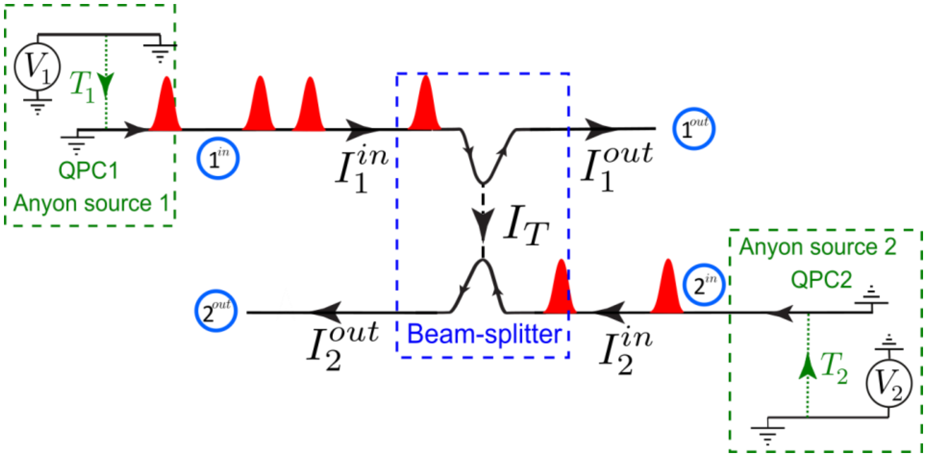
Balanced collider, $I_1^{in} = I_2^{in}$, electron case, $\nu = 2$

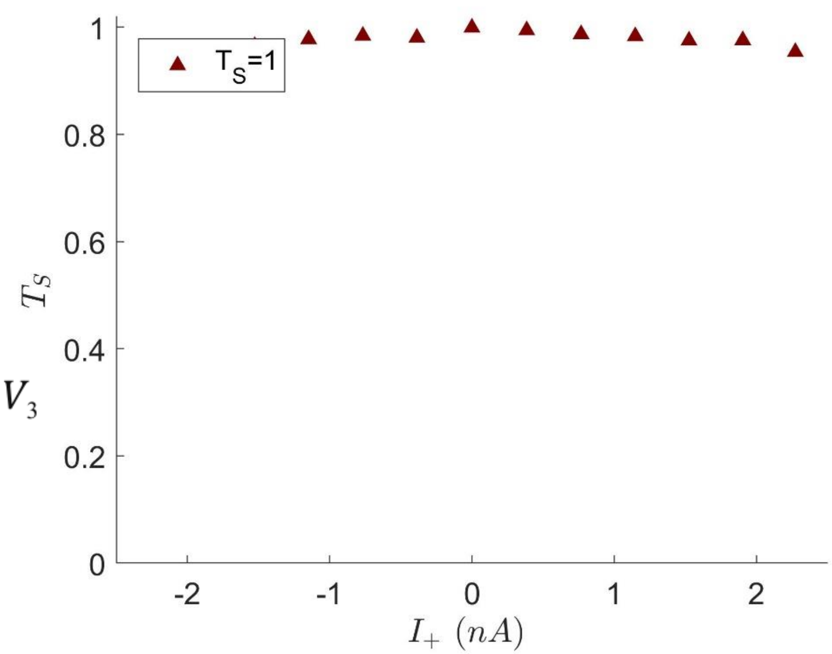
Integer case: q $= e$, fermions

$\nu = 2, T = 0.4, T_S = 1$



$\langle \delta V_3 \delta V_4 \rangle = \gamma S_{I_3 I_4}$





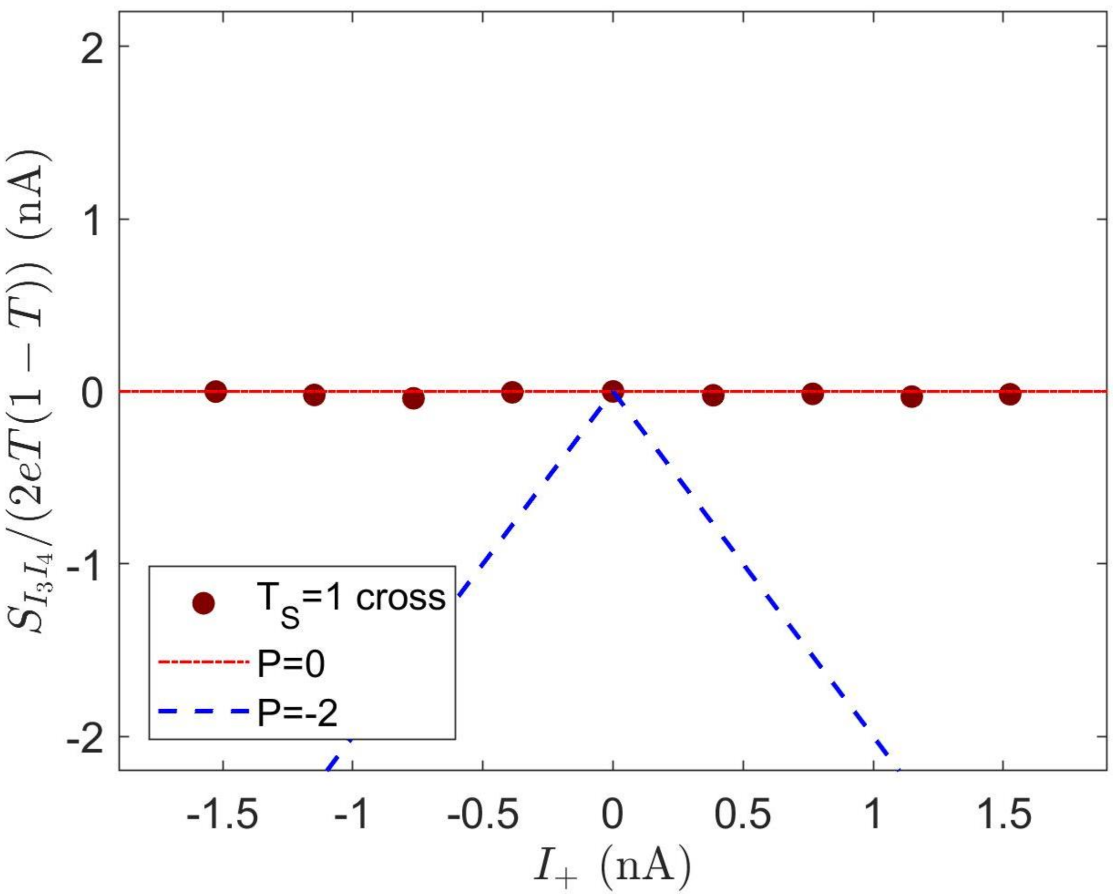
H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

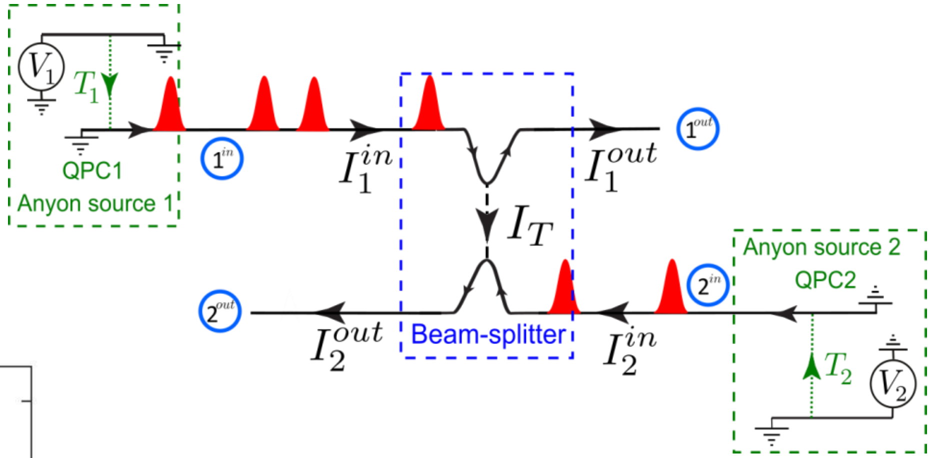
M. Ruelle et al., PRX **13**, 011031 (2023)

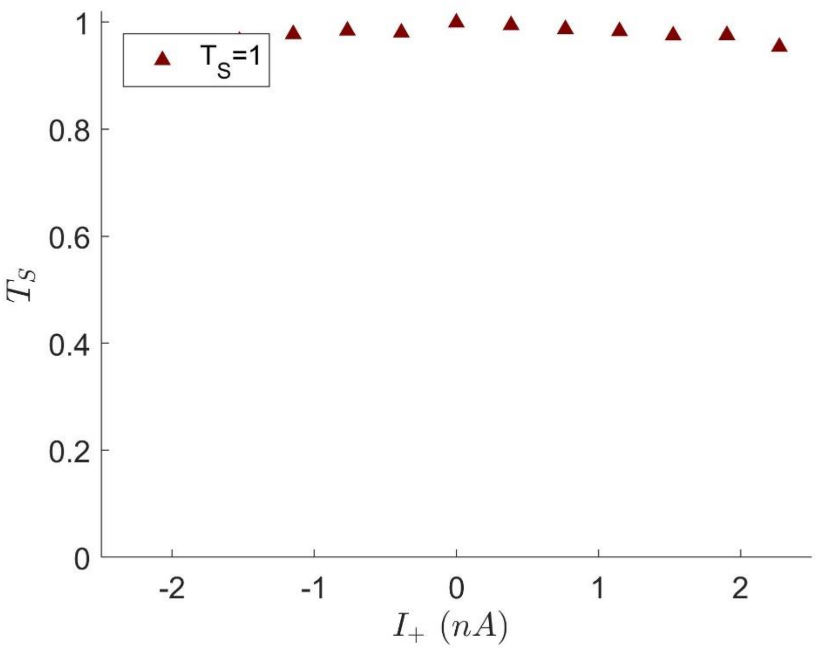
Balanced collider, $I_1^{in} = I_2^{in}$, electron case, $\nu = 2$

Integer case: $\mathrm{q} = e$, fermions

$\nu = 2, T = 0.4, T_S = 1$







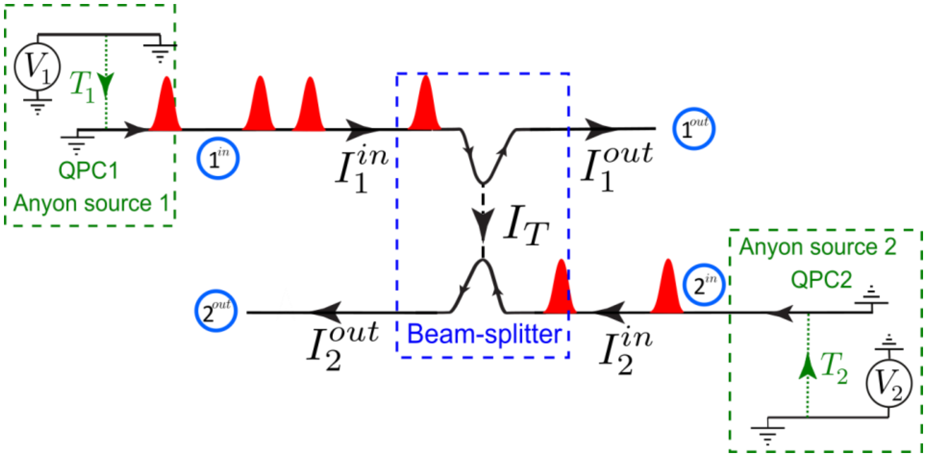
H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

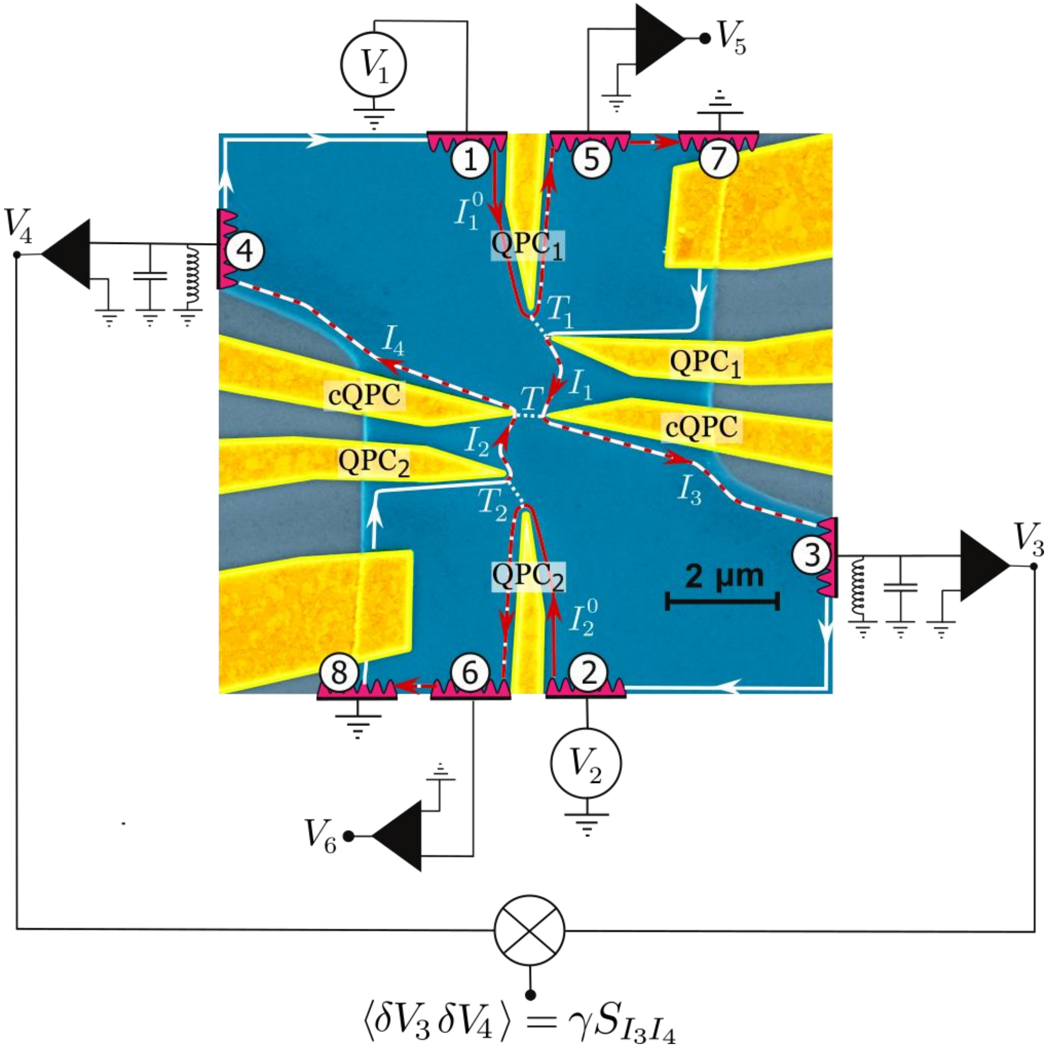
M. Ruelle et al., PRX **13**, 011031 (2023)

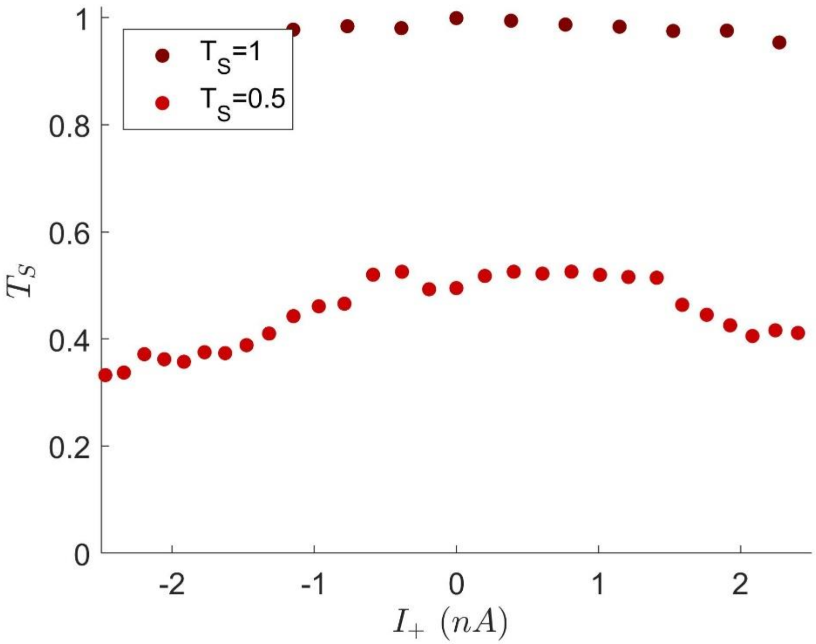
Balanced collider, $I_1^{in} = I_2^{in}$, electron case, $\nu = 2$

Integer case: q = e, fermions

$\nu = 2, T = 0.4, T_S = 1$







H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

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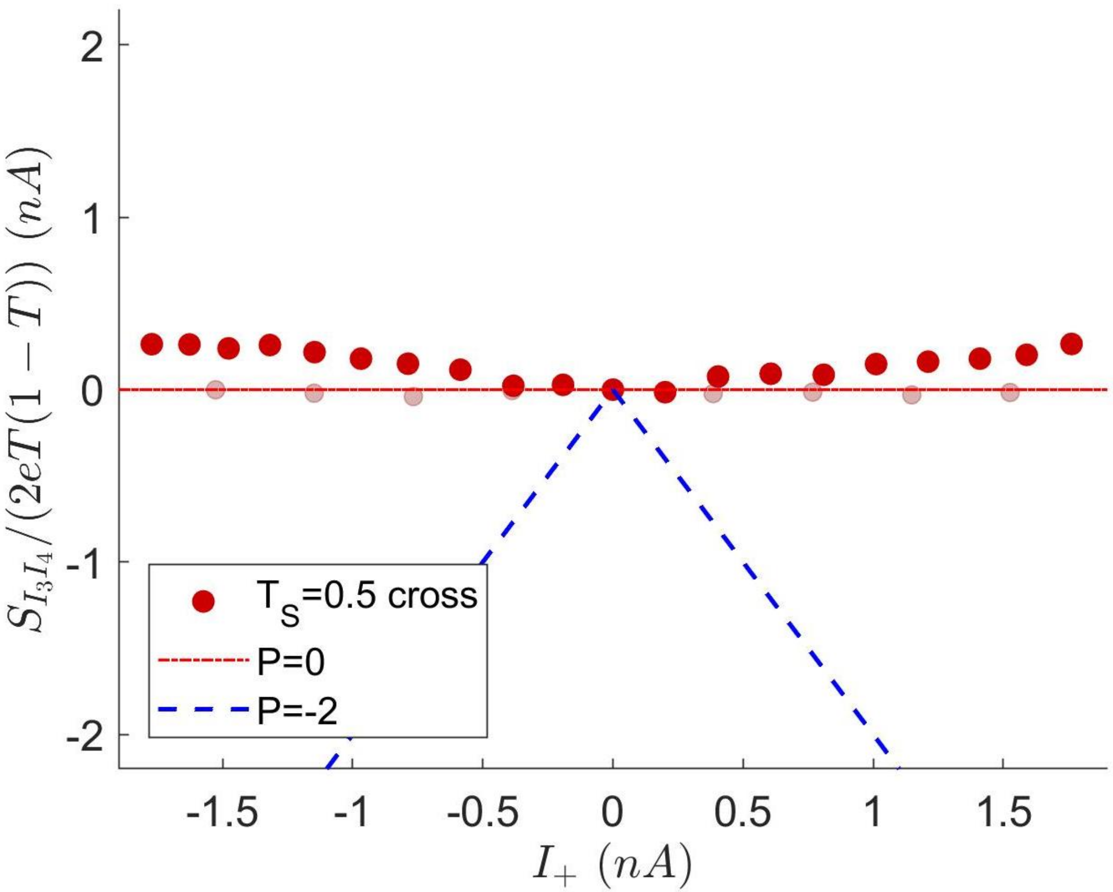

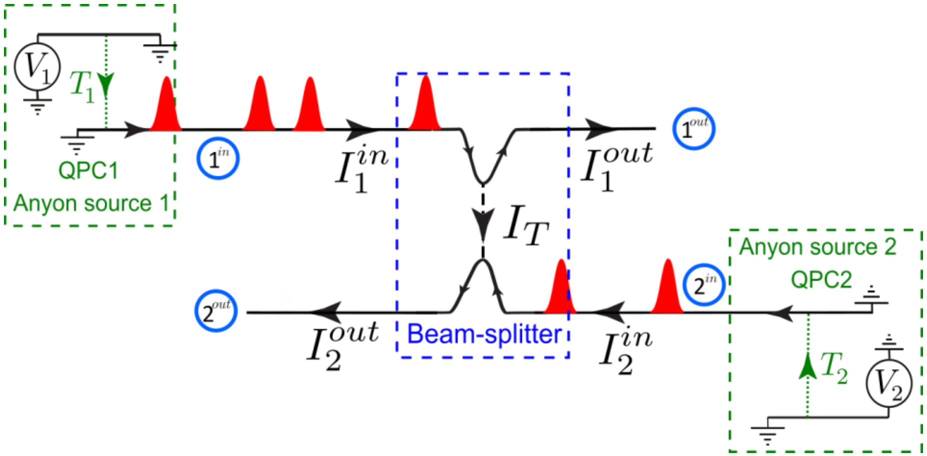

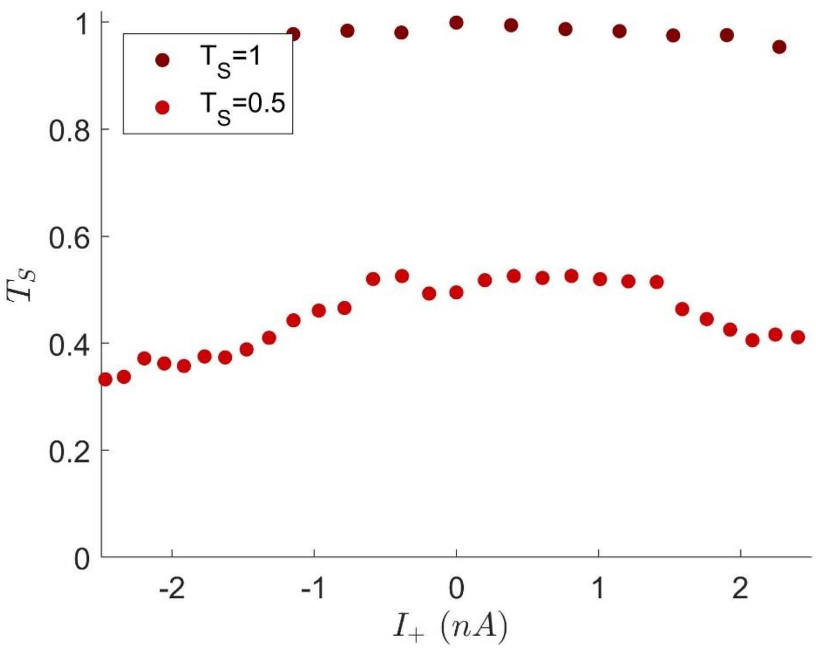

H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

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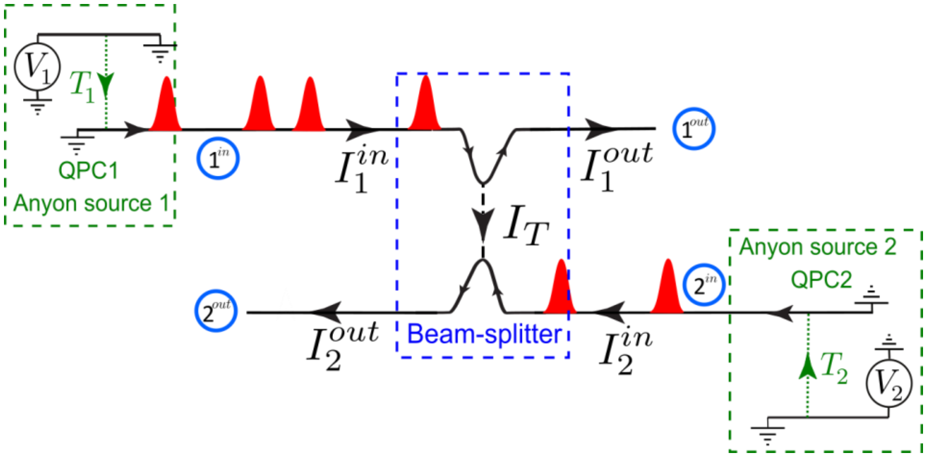
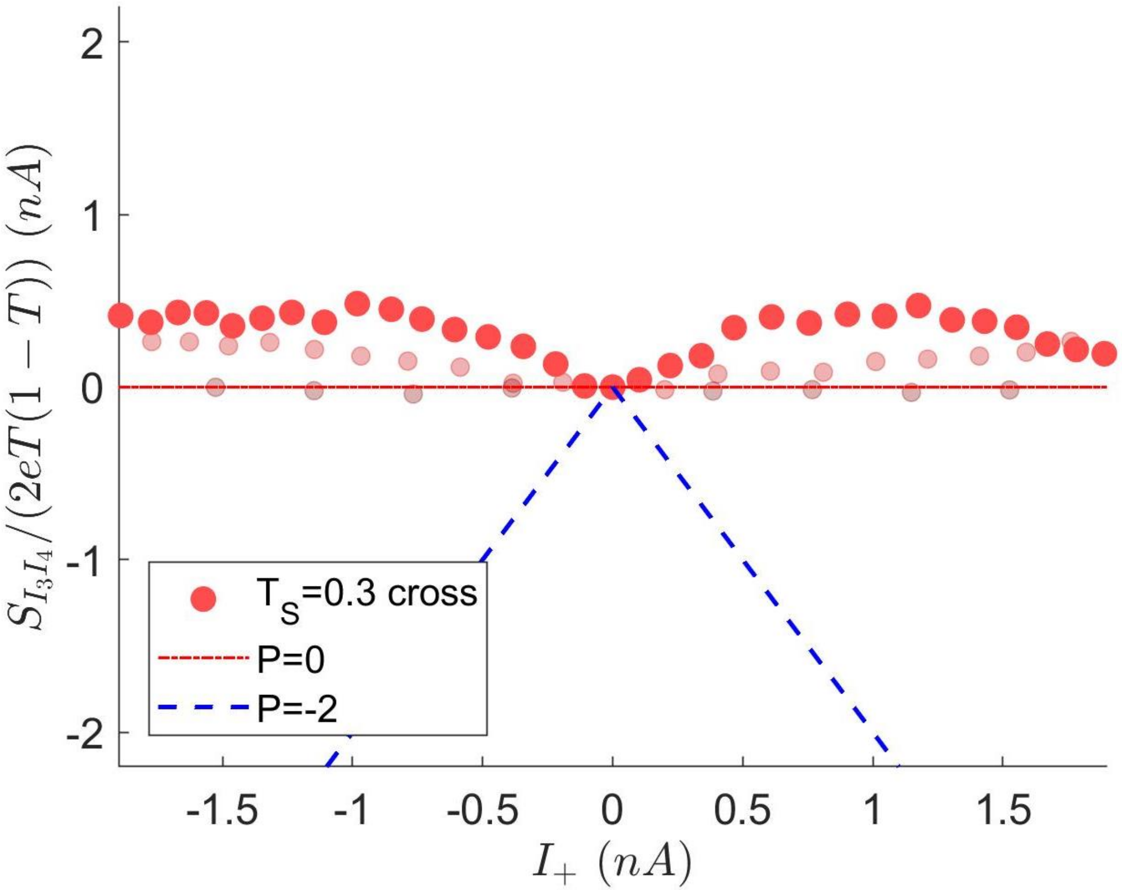
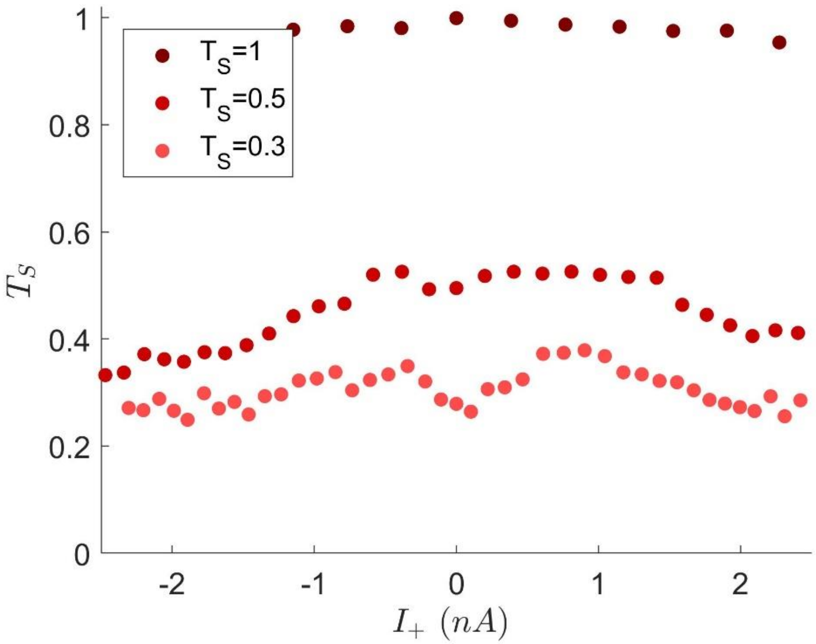






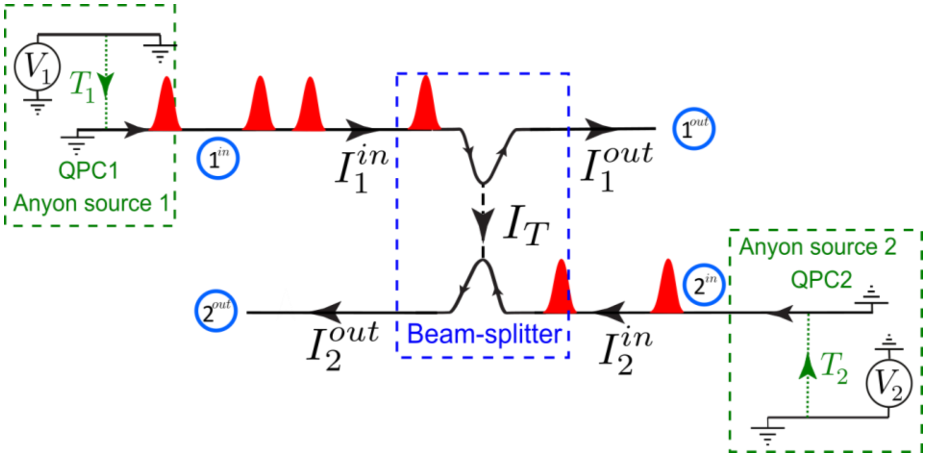
H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

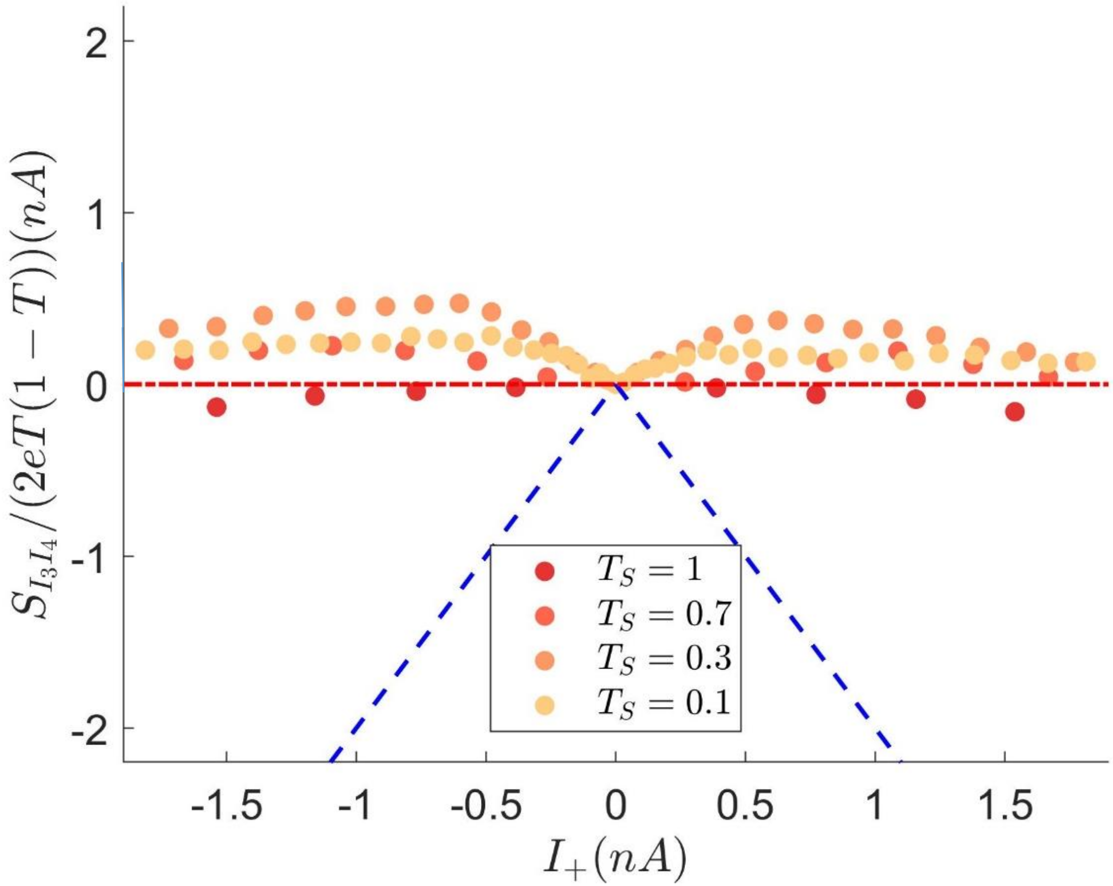
M. Ruelle et al., PRX **13**, 011031 (2023)

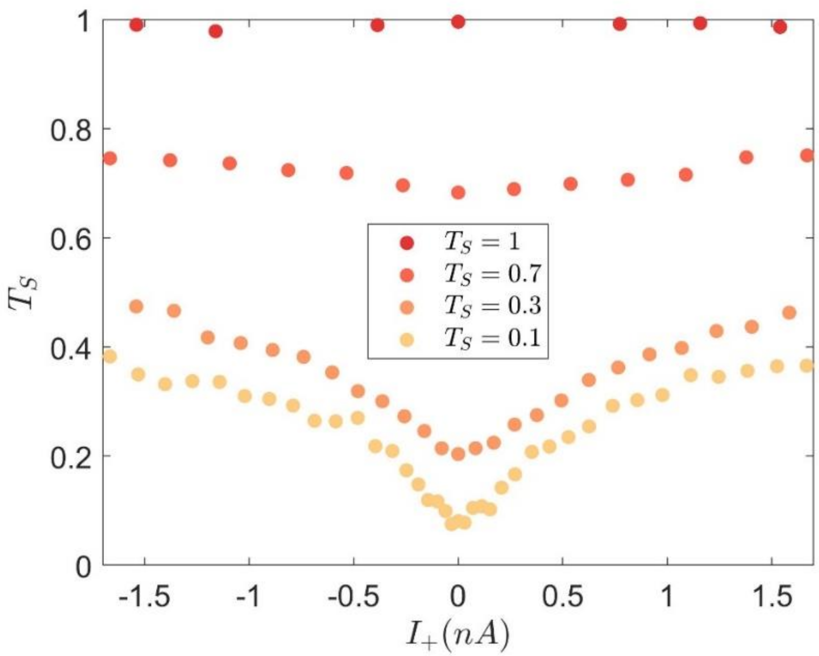
Balanced collider, $I_1^{in} = I_2^{in}$, electron case, $\nu = 3$

Integer case: q $= e$, fermions

$\nu = 3, T = 0.4, T_S = 1, 0.7, 0.3, 0.1$







$P\left(I_1^{in} = I_2^{in}\right) = 0^+$ fermions

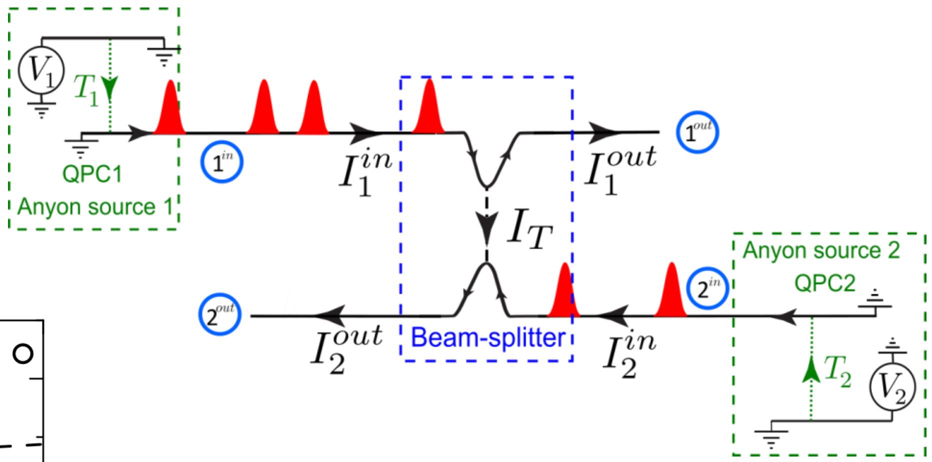
H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

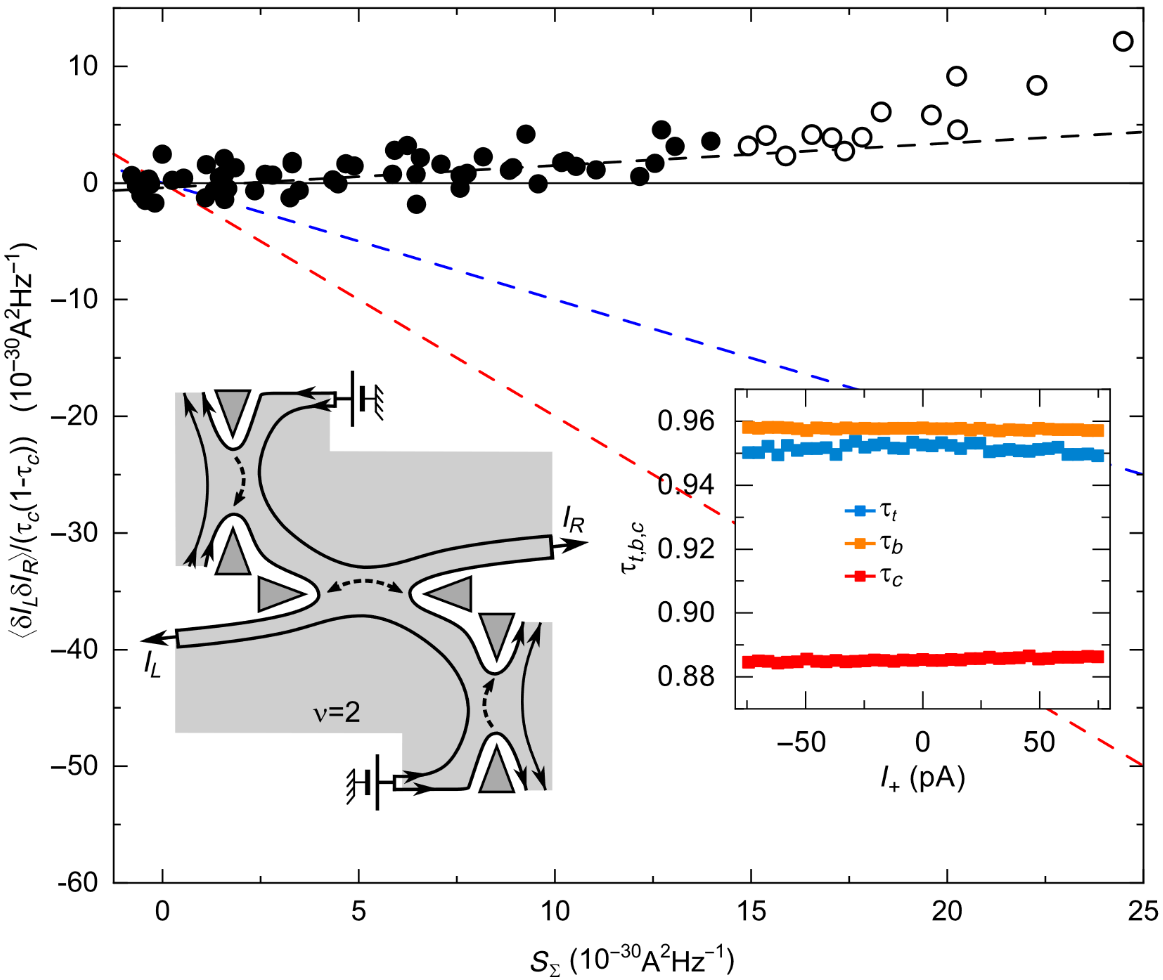
M. Ruelle et al., PRX **13**, 011031 (2023)

Balanced collider, $I_1^{in} = I_2^{in}$, electron case, $\nu = 2$

Integer case: q $= e$, electrons

$\nu = 2, \nu = 3$



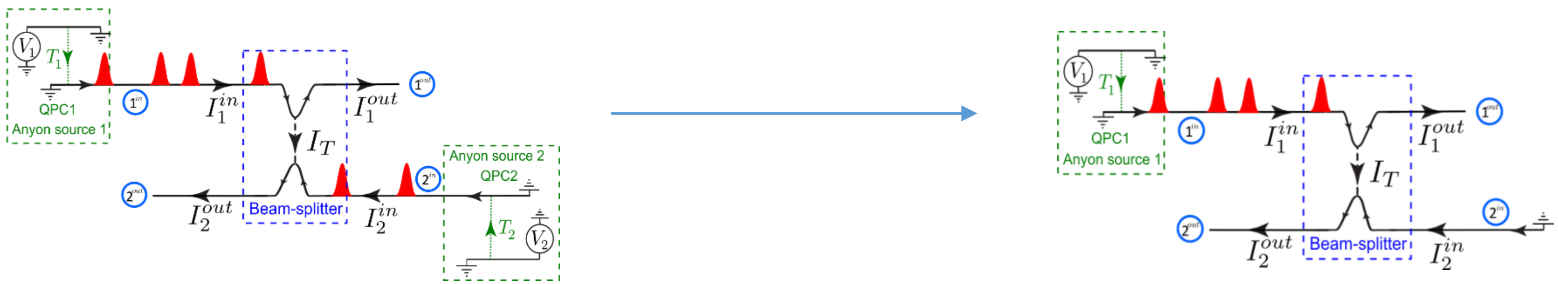


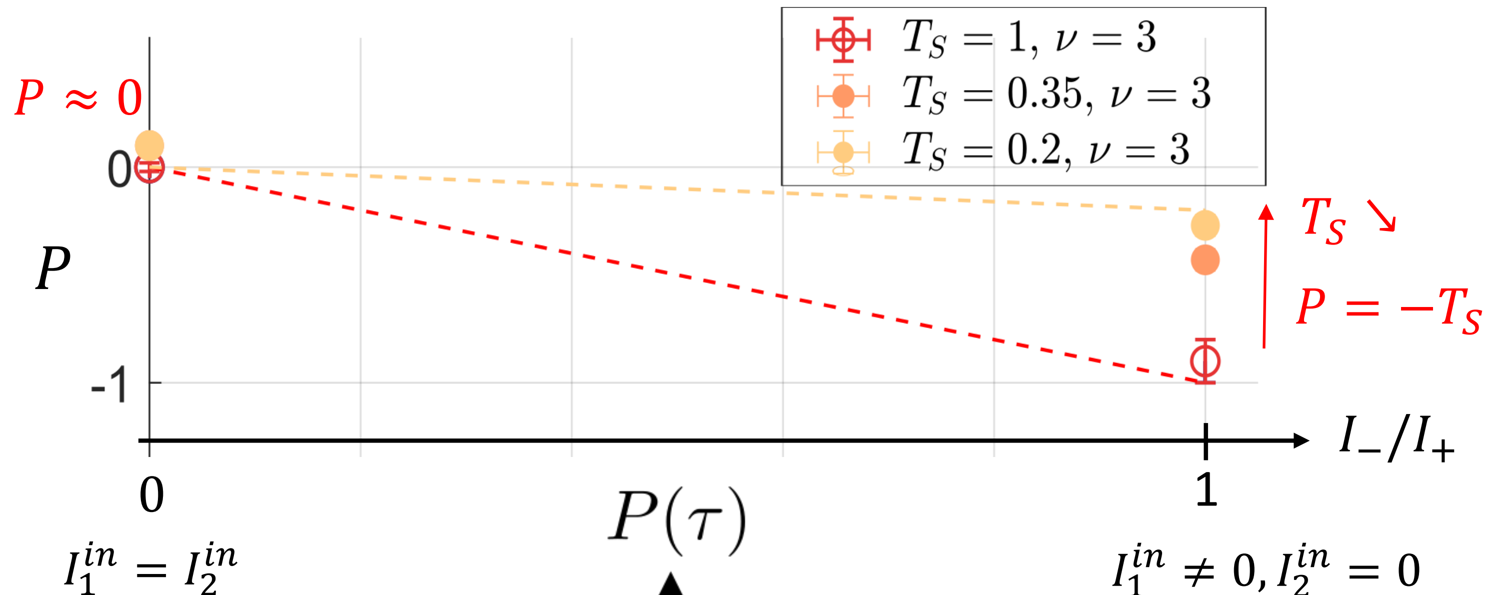
$$P\left(I_1^{in} = I_2^{in}\right) = 0^{+} \text{ fermions}$$

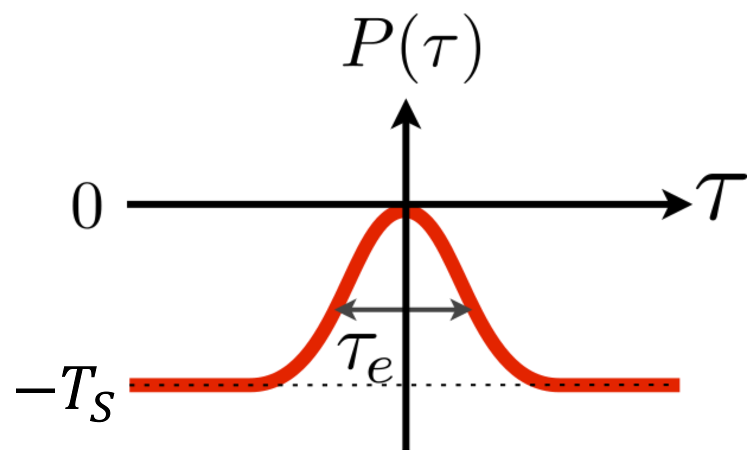
Other experiment in
F. Pierre and A. Anthore group

P. Glidic et al., Phys. Rev. X **13**, 011030 (2023).

Anyon/Fermion collisions, from balanced to unbalanced case







H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

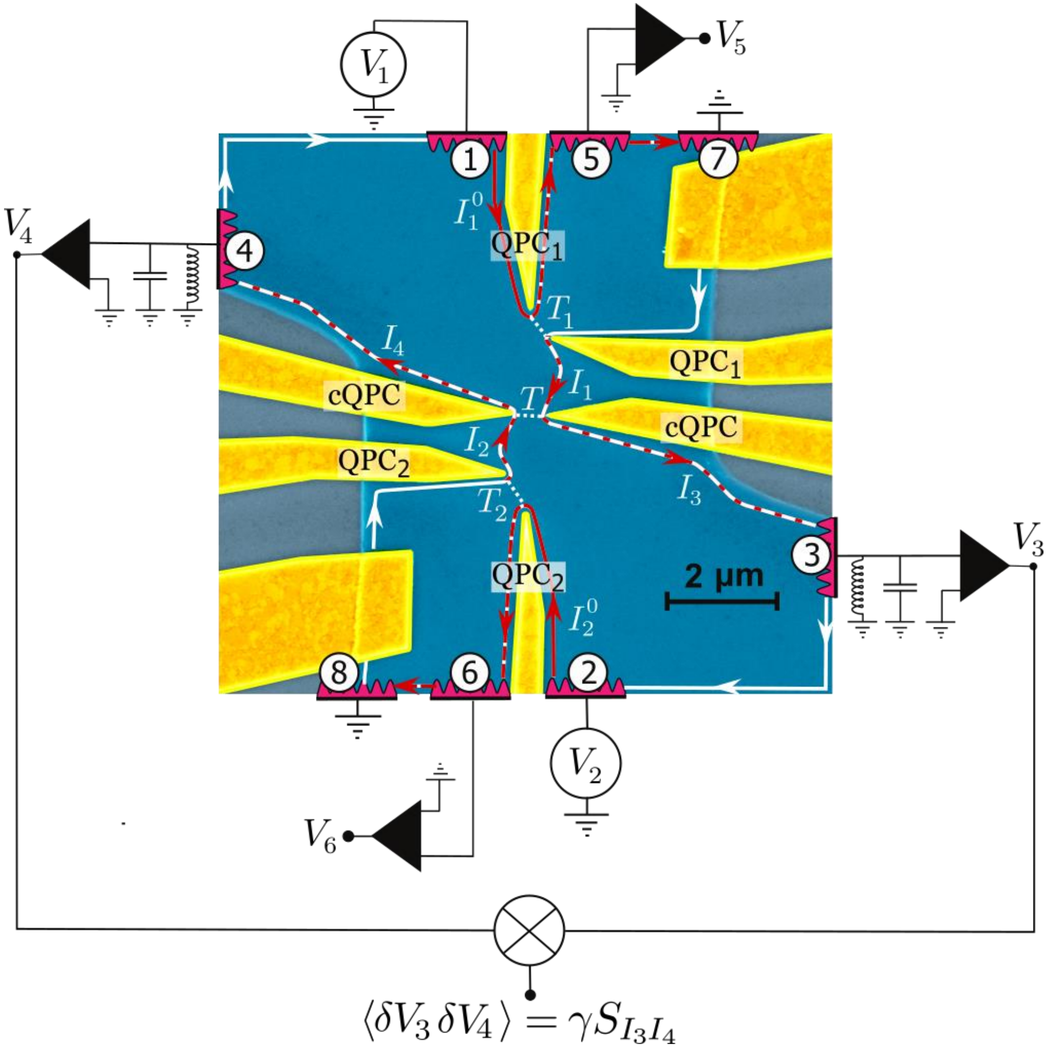
M. Ruelle et al., PRX **13**, 011031 (2023)

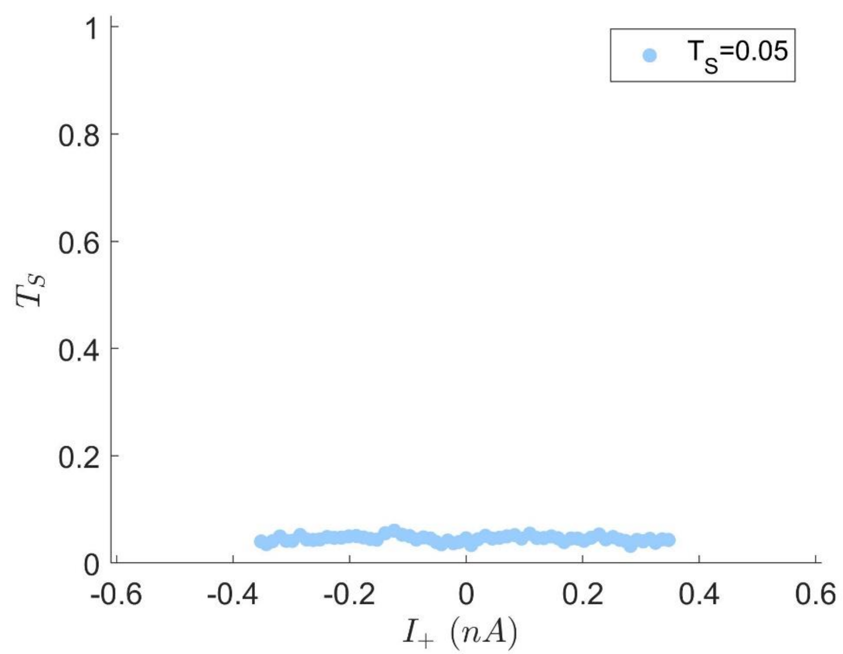
Balanced collider, $I_1^{in} = I_2^{in}$, anyon case, $\nu = 1/3$

Balanced case: $I_1^{in} = I_2^{in}$

Fractional case: q $= e/3$, anyons

$\nu = 1/3, T = 0.3, T_S = 0.05$





H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

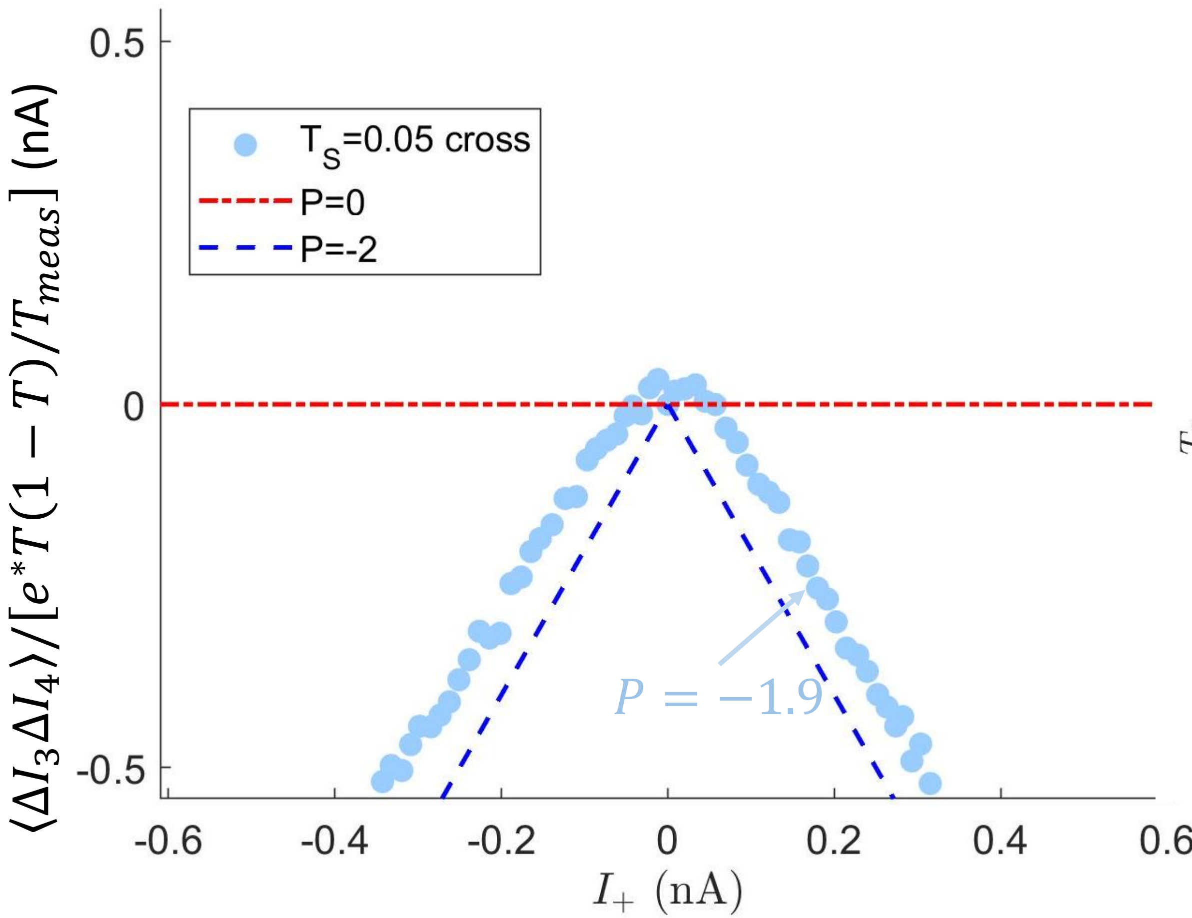
M. Ruelle et al., PRX **13**, 011031 (2023)

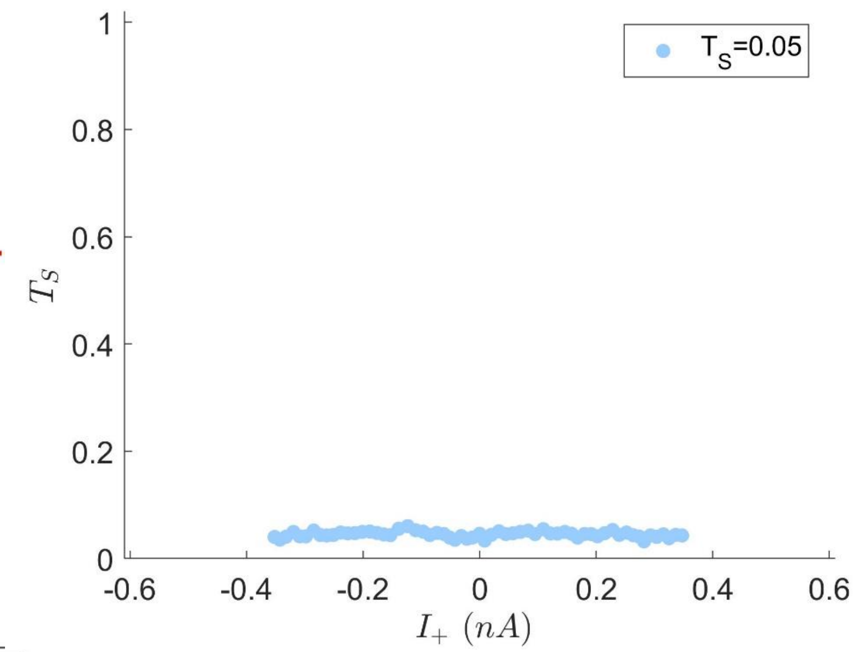
Balanced collider, $I_1^{in} = I_2^{in}$, anyon case, $\nu = 1/3$

Balanced case: $I_1^{in} = I_2^{in}$

Fractional case: q $= e/3$, anyons

$\nu = 1/3, T = 0.3, T_S = 0.05$





$P\left(I_1^{in} = I_2^{in}\right) \approx -2$ anyons ($T_S \ll 1$)

H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

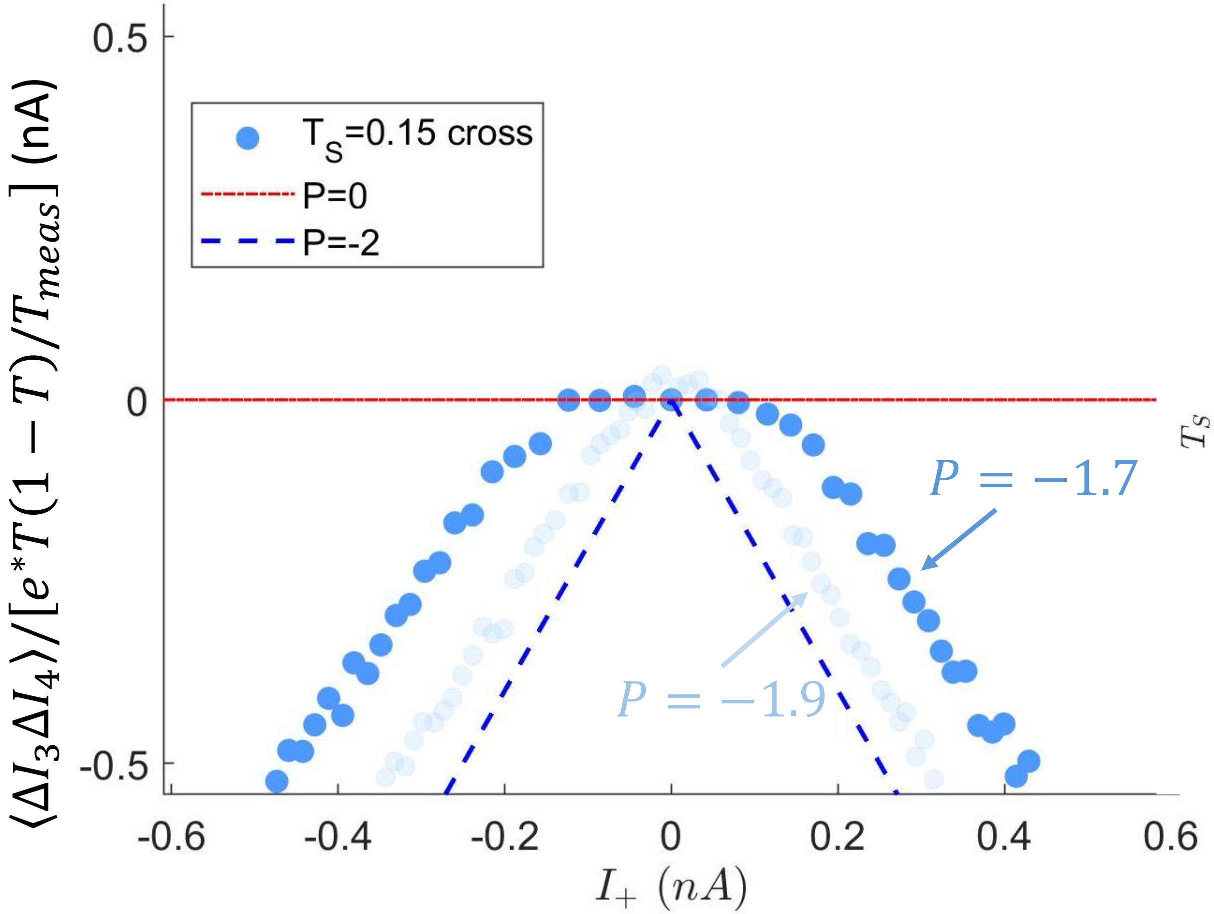
M. Ruelle et al., PRX **13**, 011031 (2023)

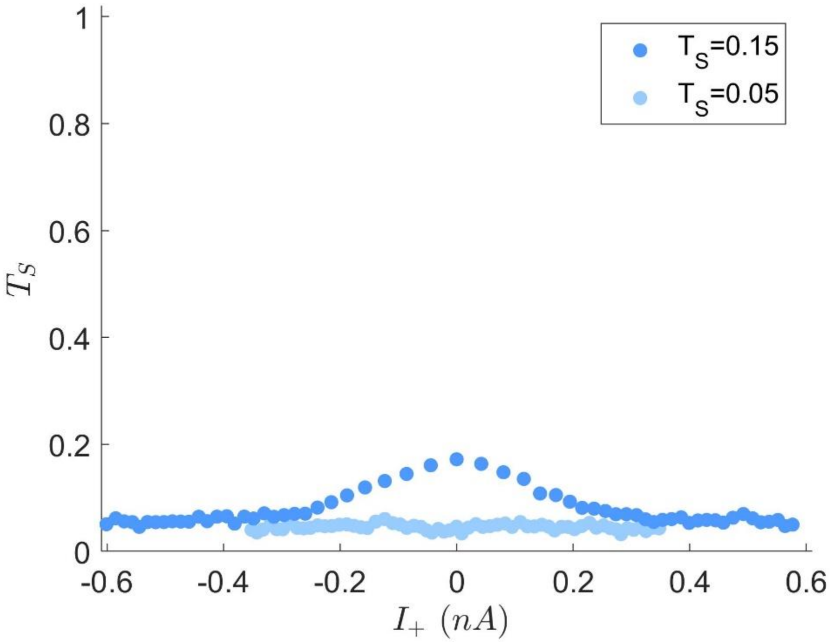
Balanced collider, $I_1^{in} = I_2^{in}$, anyon case, $\nu = 1/3$

Balanced case: $I_1^{in} = I_2^{in}$

Fractional case: $q = e/3$, anyons

$\nu = 1/3, T = 0.3, T_S = 0.15$





$P\left(I_1^{in} = I_2^{in}\right) \approx -2$ anyons ($T_S \ll 1$)

H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

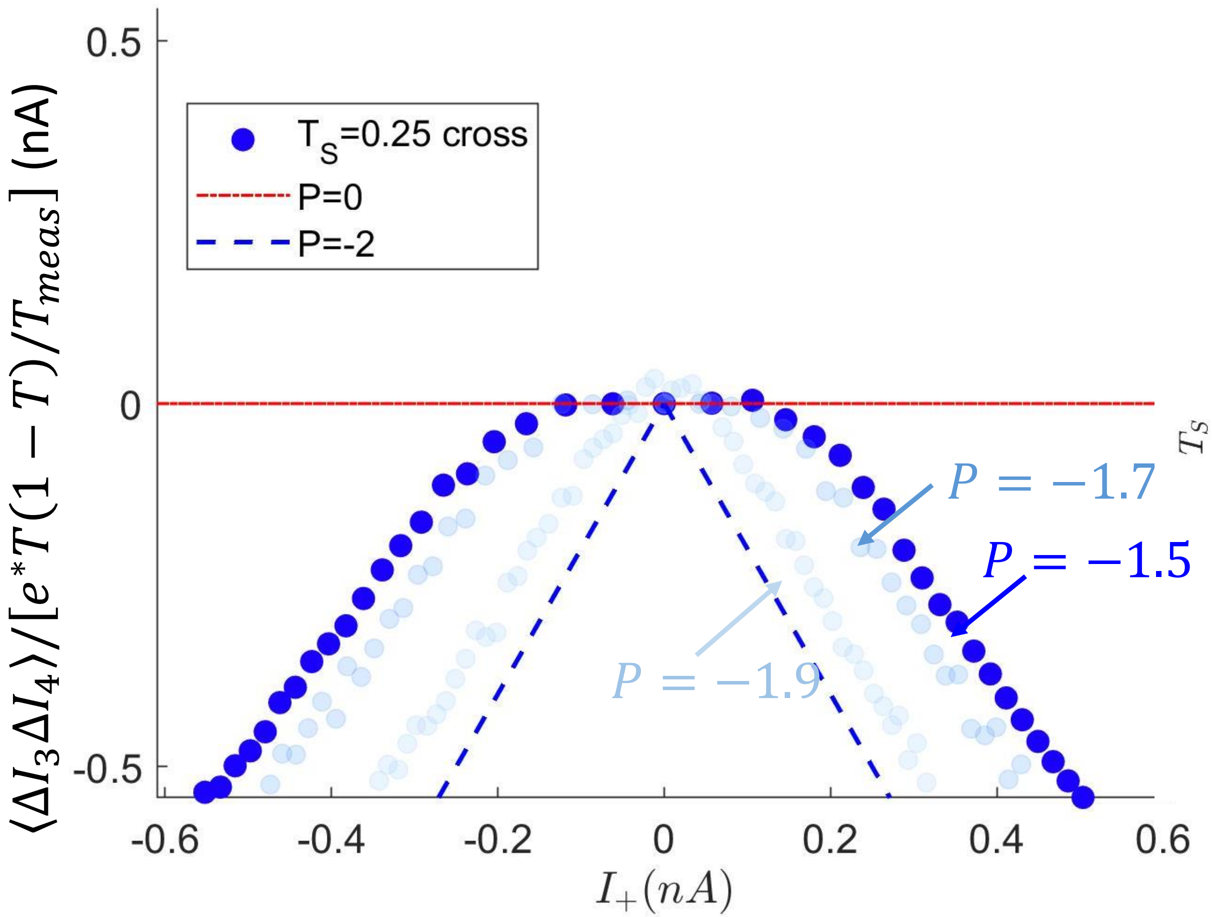
M. Ruelle et al., PRX **13**, 011031 (2023)

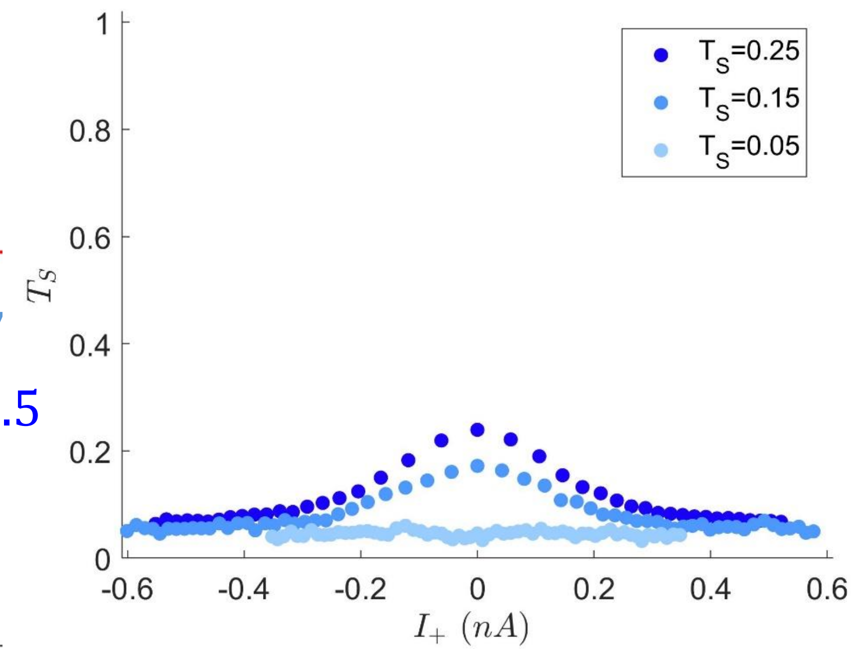
Balanced collider, $I_1^{in} = I_2^{in}$, anyon case, $\nu = 1/3$

Fractional case: q $= e/3$, anyons

$\nu = 1/3, T = 0.3, T_S = 0.25$

Balanced case: $I_1^{in} = I_2^{in}$





$P\left(I_1^{in} = I_2^{in}\right) \approx -2$ anyons $(T_S \ll 1)$

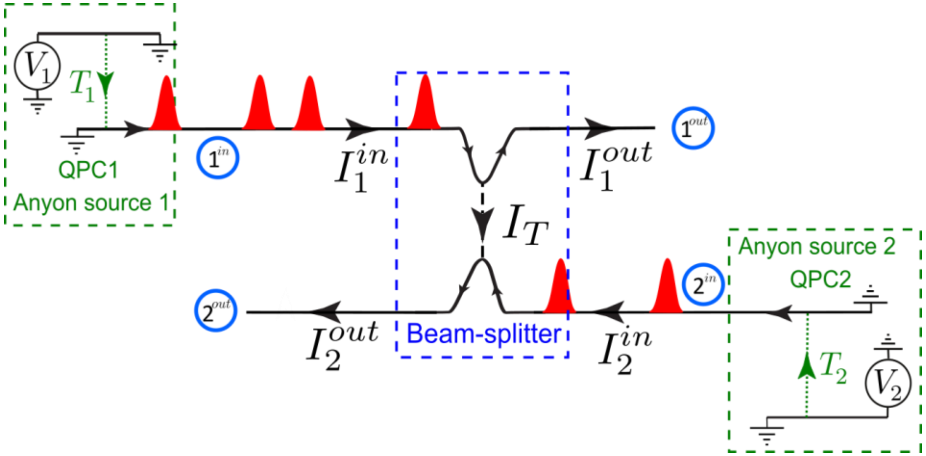
H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

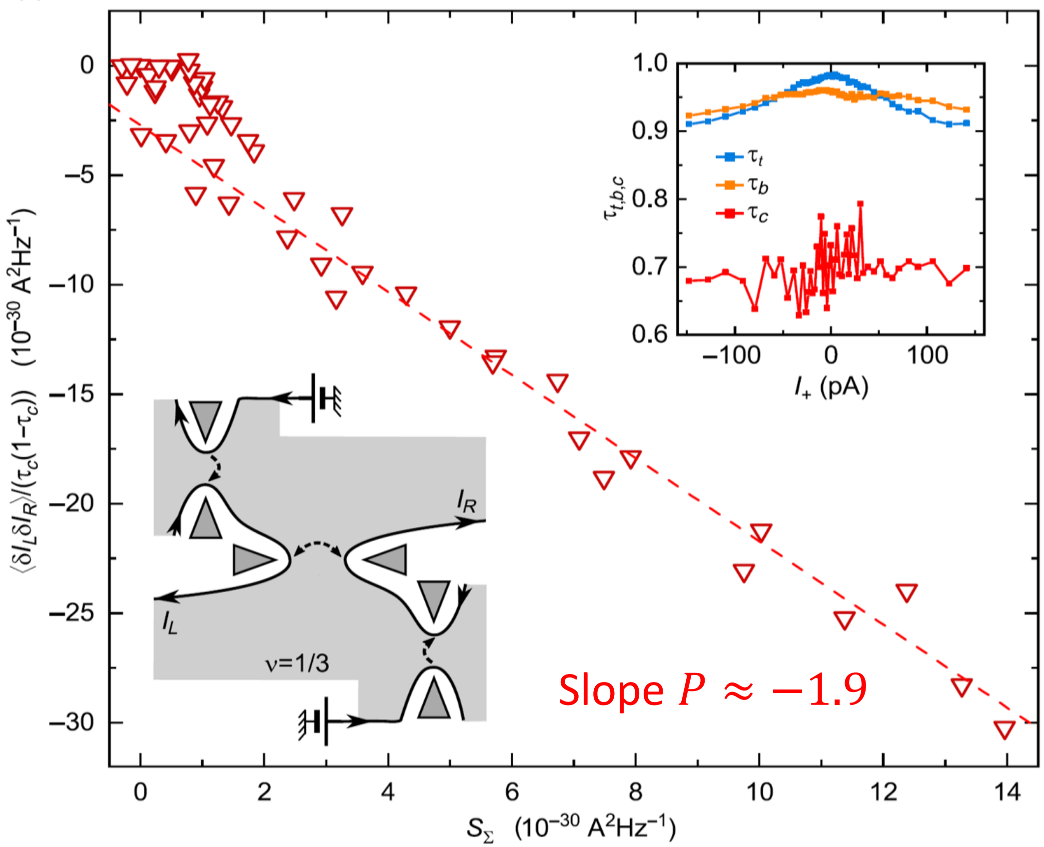
M. Ruelle et al., PRX **13**, 011031 (2023)

Balanced collider, $I_1^{in} = I_2^{in}$, anyon case, $\nu = 1/3$

Fractional case: $q = e/3$, anyons

$\nu = 1/3, T = 0.3, T_S = 0.05$





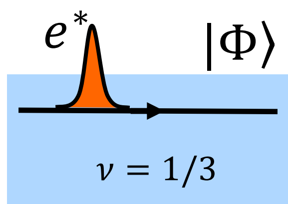
Other experiment in
F. Pierre and A. Anthore group

P. Glidic et al., Phys. Rev. X **13**, 011030 (2023).

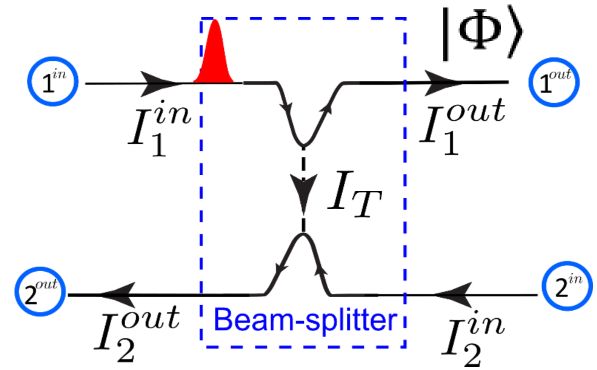
$$P\left(I_1^{in} = I_2^{in}\right) \approx -2 \text{ anyons } (T_S \ll 1)$$

Anyon tunneling at a QPC, single anyon emitted

Single anyon incoming on the QPC at time t_0:



$$|\Phi\rangle = \psi_r^+(t_0)|GS\rangle$$



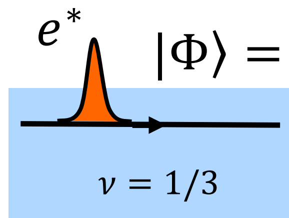
Tunneling rate: $$\Gamma_{2\to 1} \propto 2Re\left[\int_0^{+\infty} d\tau \underbrace{\left\langle \Phi|\psi_1^{in}(\tau)\psi_1^{+,in}(0)|\Phi\right\rangle}_{\text{Non-equilibrium: 1 anyon emitted}} \underbrace{\left\langle GS|\psi_2^{+,in}(\tau)\psi_2^{in}(0)|GS\right\rangle}_{\text{Equilibrium: } G_{eq,\delta}(\tau)}\right]$$

Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)

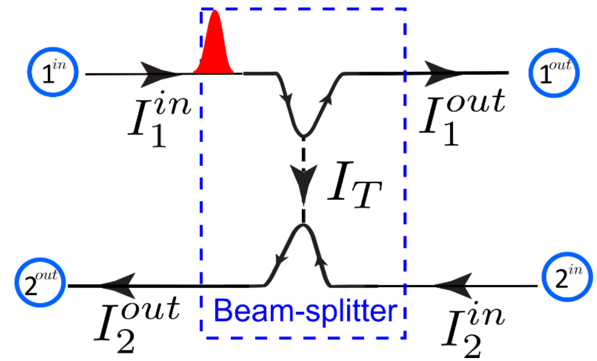
Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

Anyon tunneling at a QPC, single anyon emitted

Single anyon incoming on the QPC at time t_0:



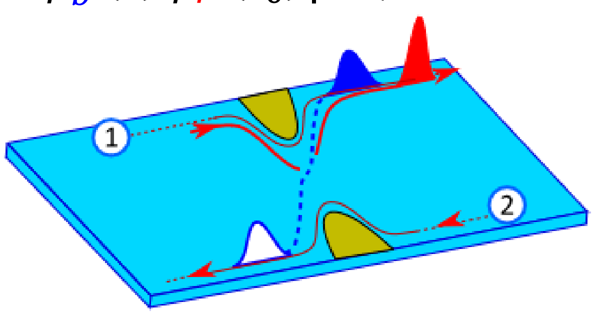
$$|\Phi\rangle = \psi_r^+(t_0)|GS\rangle$$



Tunneling rate: $$\Gamma_{2\to 1} \propto 2Re\left[\int_0^{+\infty} dt' \langle GS|\psi_r(t_0)\psi_b(\tau)\psi_b^+(0)\psi_r^+(t_0)|GS\rangle\langle GS|\psi_{2,a}^+(\tau)\psi_{2,a}(0)|GS\rangle\right]$$

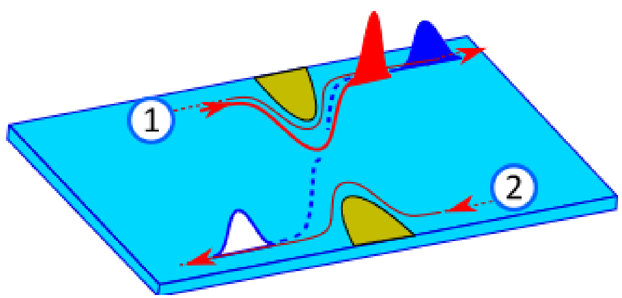
$0 \le t_0 \le \tau$

$\psi_b^+(\tau)\psi_r^+(t_0)|GS\rangle$



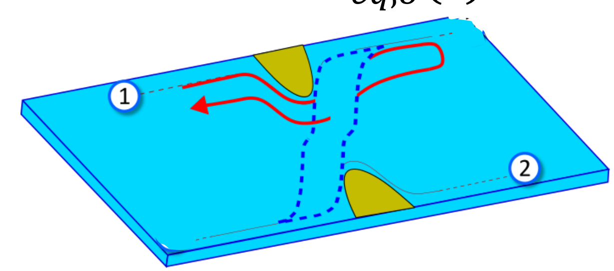
tunnels at $\tau > t_0$

$\psi_b^+(0)\psi_r^+(t_0)|GS\rangle$



tunnels at $0 < t_0$

$$\langle\Phi|\psi_{1,a}(\tau)\psi_{1,a}^+(0)|\Phi\rangle = e^{-i\theta_{rb}}G_{eq,\delta}(\tau)$$



θ_{rb}: mutual braiding phase between blue and red anyons

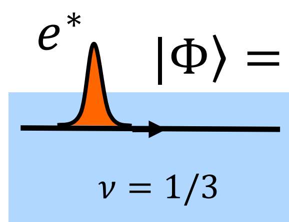
$\nu = 1/3 \qquad \theta = 2\pi/3$

Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)

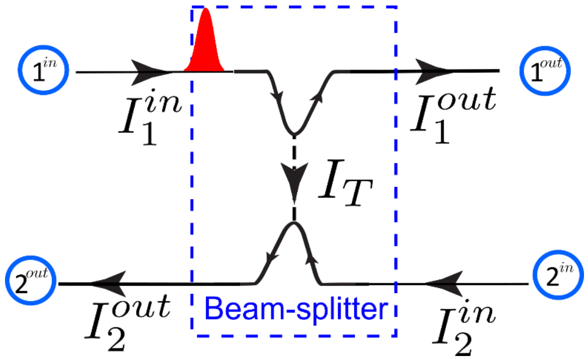
Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

Anyon tunneling at a QPC, single anyon emitted

Single anyon incoming on the QPC at time t_0:



$$|\Phi\rangle = \psi_r^+(t_0)|GS\rangle$$

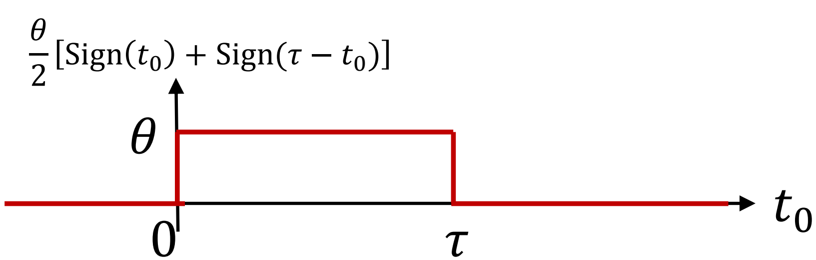


Tunneling rate: $$\Gamma_{2\to1} \propto 2Re\left[\int_0^{+\infty} dt' \langle GS|\psi_r(t_0)\psi_b(\tau)\ \psi_b^+(0)\psi_r^+(t_0)|GS\rangle\langle GS|\psi_{2,a}^+(\tau)\psi_{2,a}(0)|GS\rangle\right]$$

$$\underline{0 \le t_0 \le \tau}$$

Anyons at the edge: $$\psi_a^+(x)\psi_a^+(x') = e^{-i\frac{\theta}{2}\,\mathrm{Sign}(x-x')}\psi_a^+(x')\psi_a^+(x)$$

$$\Gamma_{2\to1} \propto 2Re\left[\int_0^{+\infty} d\tau\ e^{-i\frac{\theta}{2}[\mathrm{Sign}(t_0)+\mathrm{Sign}(\tau-t_0)]}\langle GS|\psi_b(\tau)\ \psi_b^+(0)|GS\rangle\langle GS|\psi_{2,a}^+(\tau)\psi_{2,a}(0)|GS\rangle\right]$$



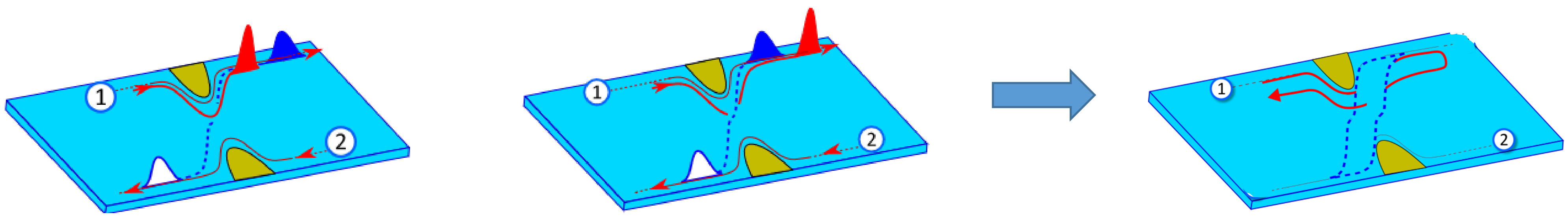
$$\Gamma_{2\to1} \propto 2Re\left[\int_0^{+\infty} e^{-i\theta N_1(\tau)} G_\delta(\tau)^2 d\tau\right]$$

Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)

Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

Anyon tunneling at a QPC, random anyon source

$N_1(t,t')$ anyons incoming on the QPC between times t' and t



tunnels at $t = 0 < t_0$

tunnels at $\tau > t_0$

mutual braiding phase

$$\Gamma_{2\to1} \propto 2Re\left[\int_0^{+\infty} e^{-i\theta N_1(\tau)} G_\delta(\tau)^2 d\tau\right]$$

number of anyons emitted
at input 1 reaching the QPC

equilibrium Green's function,
long-time decay governed by δ

$$\Gamma_{1\to2} \propto 2Re\left[\int_0^{+\infty} e^{+i\theta N_1(\tau)} G_\delta(\tau)^2 d\tau\right]$$

N_1 is a random Poissonian variable:

$$\left\langle e^{\pm i\theta N_1}\right\rangle = e^{-\langle N_1\rangle(1-e^{\pm i\theta})}$$

$$\left\langle e^{\pm i\theta N_1}\right\rangle = e^{-\frac{I_1\tau}{e^*}(1-e^{\pm i\theta})}$$

Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)
Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

Anyon tunneling at a QPC, random anyon source

$$\mathit{\Gamma}_{1\to2} \propto 2Re\left[\int_0^{+\infty} d\tau e^{-\frac{|I_1|\tau}{e^*}(1-e^{i\theta})} e^{-\frac{|I_2|\tau}{e^*}(1-e^{-i\theta})}\left[G_{eq,\delta}(\tau)\right]^2\right] = 2Re\left[\int_0^{+\infty} d\tau e^{-\frac{I_+\tau}{e^*}(1-\cos\theta)} e^{i\frac{I_-\tau}{e^*}\sin\theta}\left[G_{eq,\delta}(\tau)\right]^2\right]$$

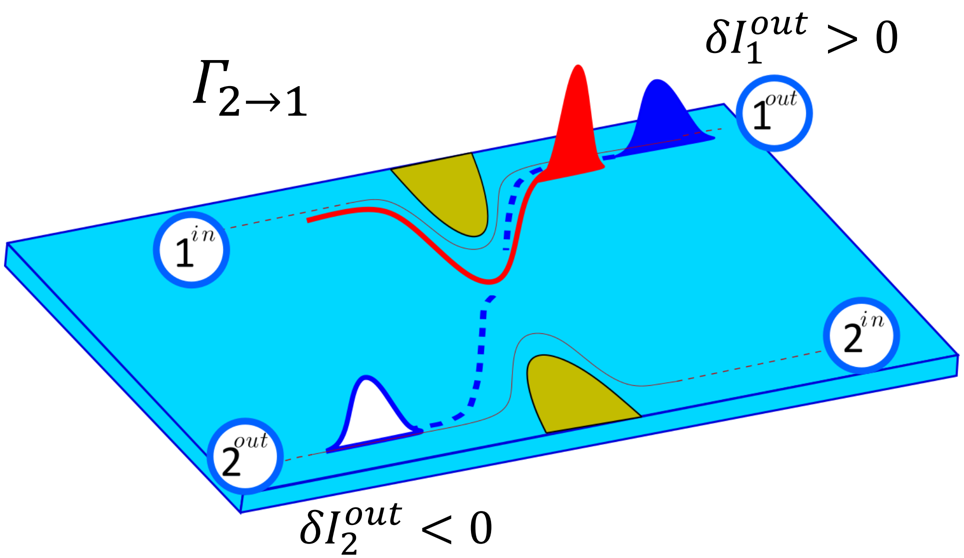
$$I_+ = I_1^{in} + I_2^{in} \qquad I_- = I_1^{in} - I_2^{in}$$

$$\mathit{\Gamma}_{1\to2} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta+\xi_+-i\xi_-)}{\Gamma(1-\delta+\xi_+-i\xi_-)}\right] \approx 2Re\left[e^{-i\pi\delta}(\xi_+-i\xi_-)^{2\delta-1}\right] \text{ for } \xi \gg 1$$

$$\mathit{\Gamma}_{2\to1} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta+\xi_++i\xi_-)}{\Gamma(1-\delta+\xi_++i\xi_-)}\right] \approx 2Re\left[e^{-i\pi\delta}(\xi_++i\xi_-)^{2\delta-1}\right] \text{ for } \xi \gg 1$$

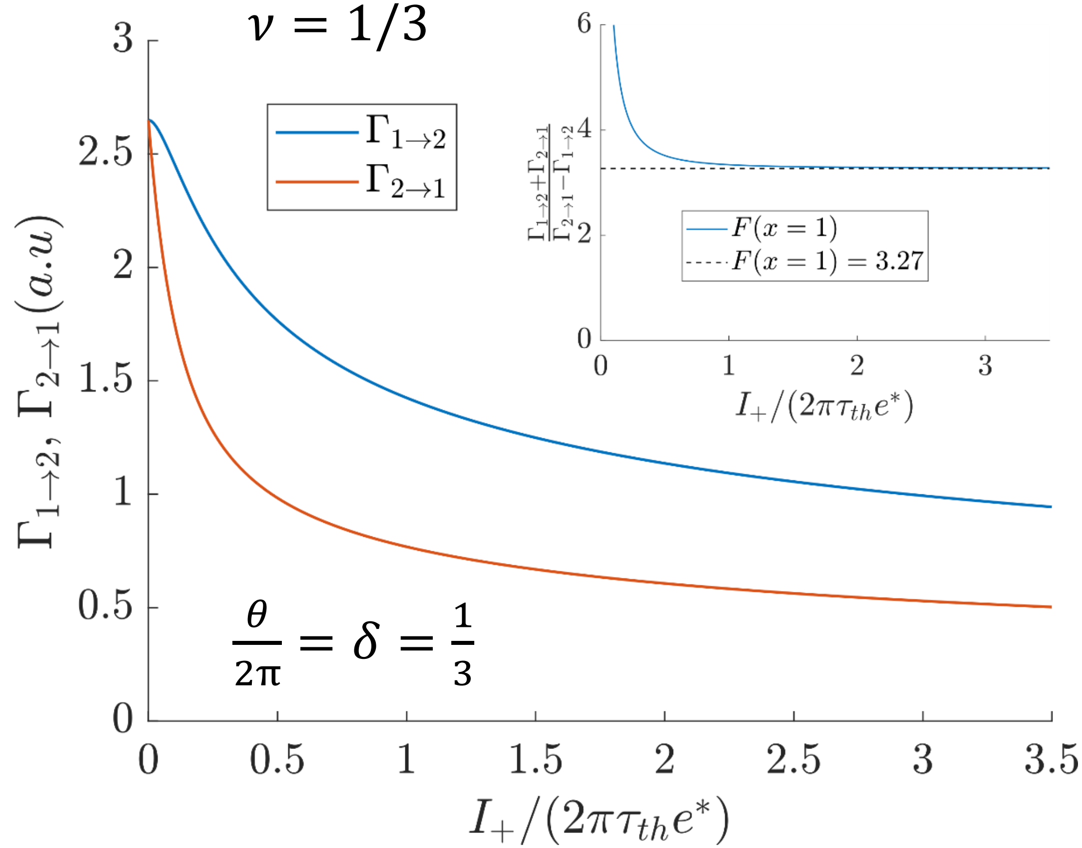
with: $\xi_+ = \dfrac{I_+}{2\pi e^*\tau_{th}}(1-\cos\theta) \qquad \xi_- = \dfrac{I_-}{2\pi e^*\tau_{th}}\sin\theta$

Interpretation, unbalanced case ($I_2^{in} = 0$)



Bunching mechanism: $S_{12}^{out} < 0$

$S_{I_T} > 2e^* I_T$



Requires only one anyon to be present

$S_{12}^{out} \propto T_S$ (and $I_1^{in} \propto T_S$)

P is a number (independent of T_S)

F is a number (independent of T_S)

This does not exist in the classical (boson and fermions) case

Anyon tunneling at a QPC, random anyon source

$$\Gamma_{1\to 2} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta+\xi_+ - i\xi_-)}{\Gamma(1-\delta+\xi_+ - i\xi_-)}\right] \approx 2Re\left[e^{-i\pi\delta}(\xi_+ - i\xi_-)^{2\delta-1}\right] \text{ for } \xi \gg 1$$

$$\Gamma_{2\to 1} \propto 2Re\left[e^{-i\pi\delta}\frac{\Gamma(\delta+\xi_+ + i\xi_-)}{\Gamma(1-\delta+\xi_+ + i\xi_-)}\right] \approx 2Re\left[e^{-i\pi\delta}(\xi_+ + i\xi_-)^{2\delta-1}\right] \text{ for } \xi \gg 1$$

with: $\xi_+ = \frac{I_+}{2\pi e^* \tau_{th}}(1-\cos\theta)$ $\quad \xi_- = \frac{I_-}{2\pi e^* \tau_{th}}\sin\theta$

$\theta/2\pi = \delta = \frac{1}{3}$

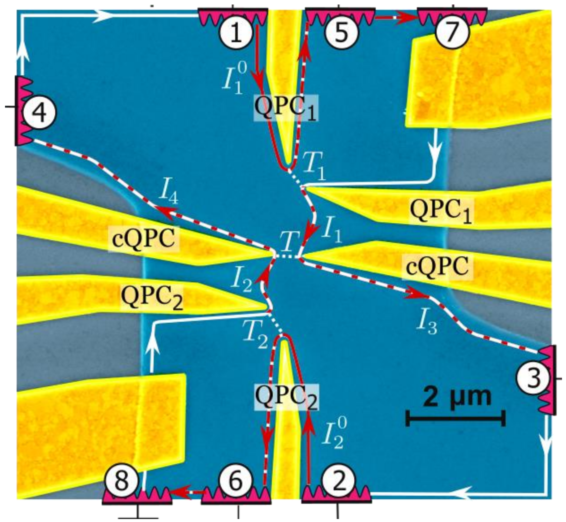
Balanced case: $I_1^{in} = I_2^{in}$ $\quad P = 1 - cot(\pi\delta)\frac{tan(\theta/2)}{1-2\delta}$ $\quad P = -2$

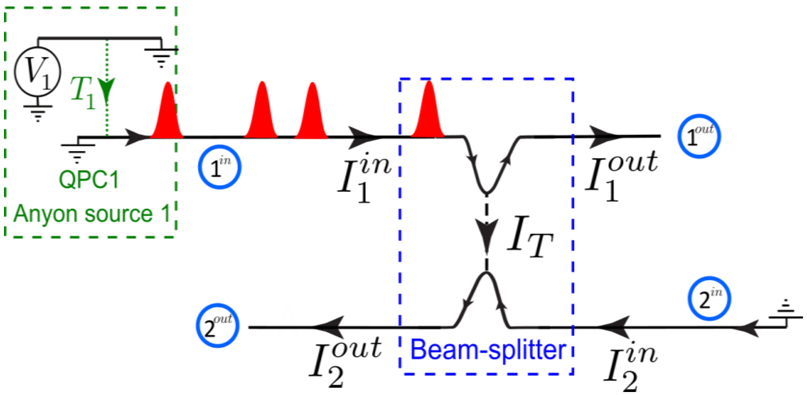
Unbalanced case: $I_1^{in} \neq 0, I_2^{in} = 0$ $\quad F = -\cot(\pi\delta)\cot\left[(2\delta-1)\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]$ $\quad F = 3.27$

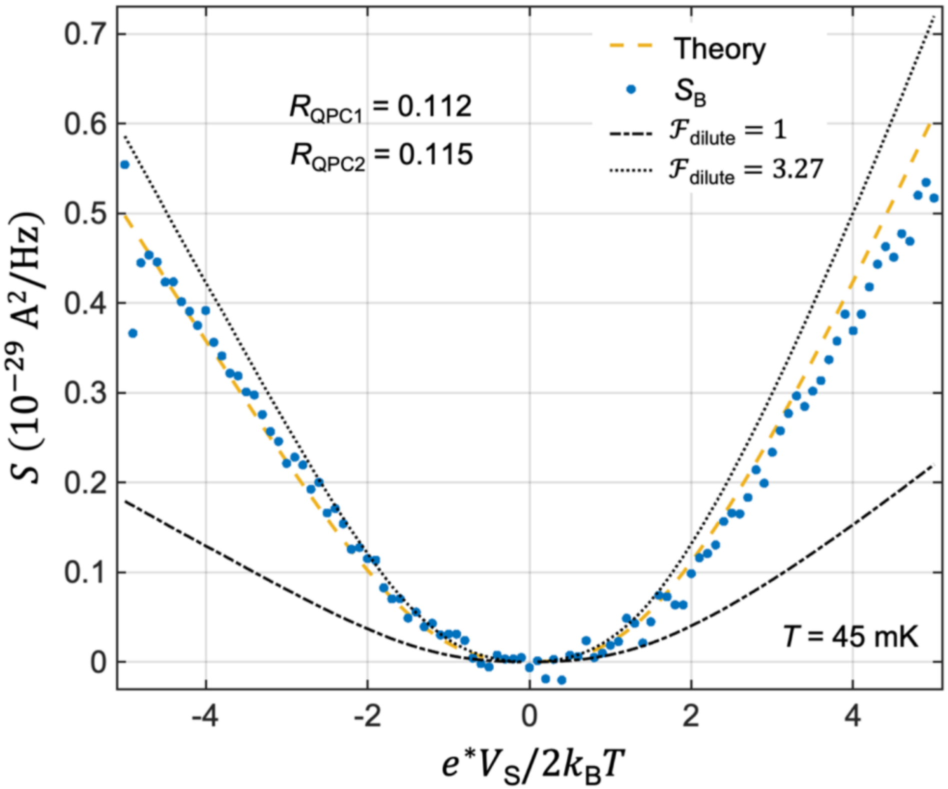
B. Rosenow, et al., PRL **116** 156802 (2016)

Lee et al., Nature (2023), https://doi.org/10.1038/s41586-023-05883-2

Unbalanced collider, $F(I_2^{in} = 0)$: Lee et al. Nature (2023)





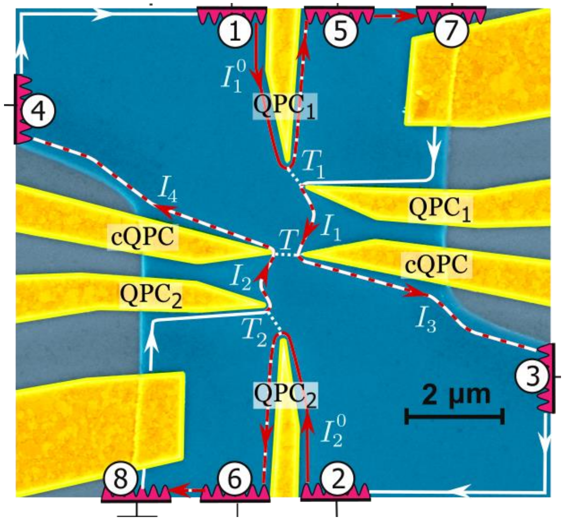


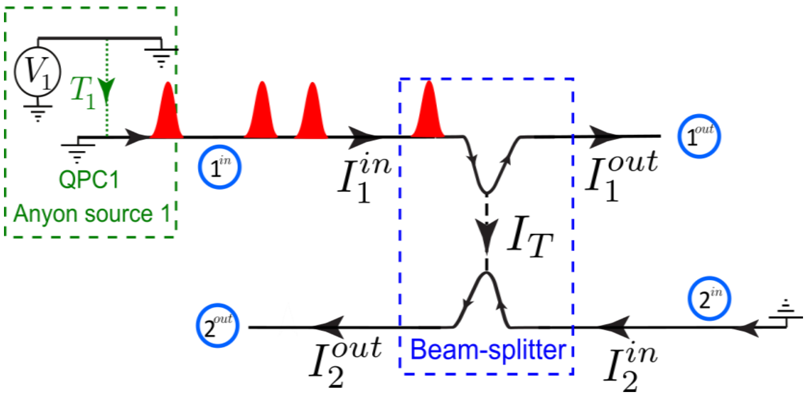
$F(I_- = I_+) \approx +3.27$

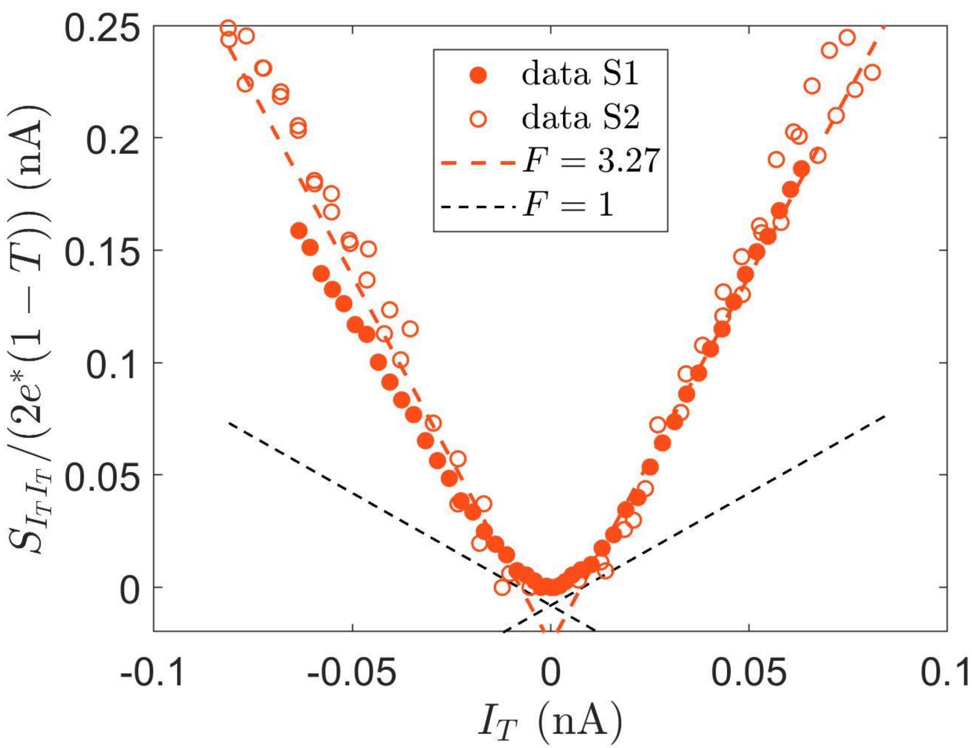
$\theta/2\pi = \delta = \frac{1}{3}$

Lee et al., Nature (2023), https://doi.org/10.1038/s41586-023-05883-2

Unbalanced collider, $\mathrm{F}(I_2^{in} = 0)$: our data



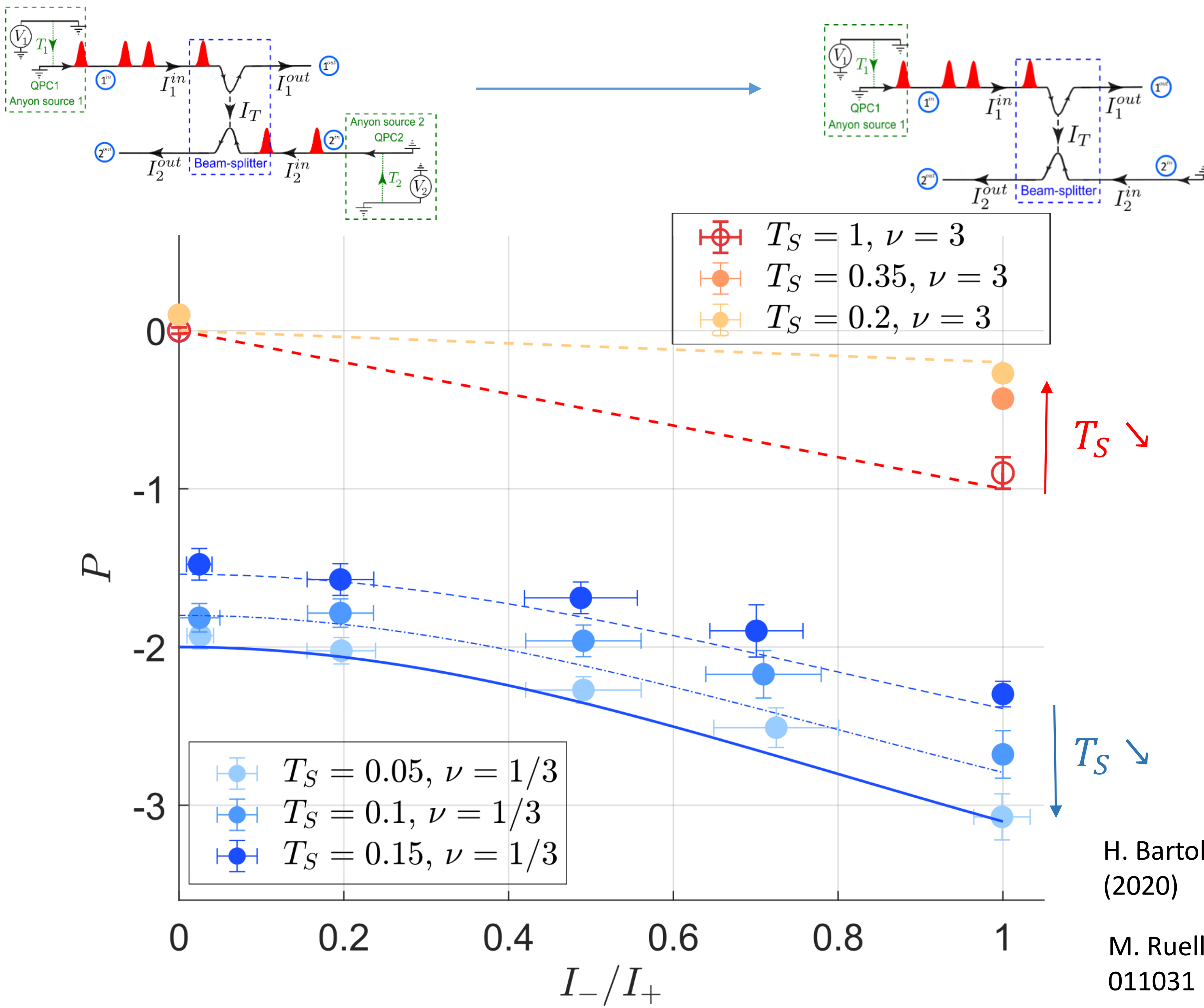




$F(I_- = I_+) \approx +3.27$

$\theta/2\pi = \delta = \frac{1}{3}$

Anyon/Fermion collisions, from the balanced to the unbalanced case



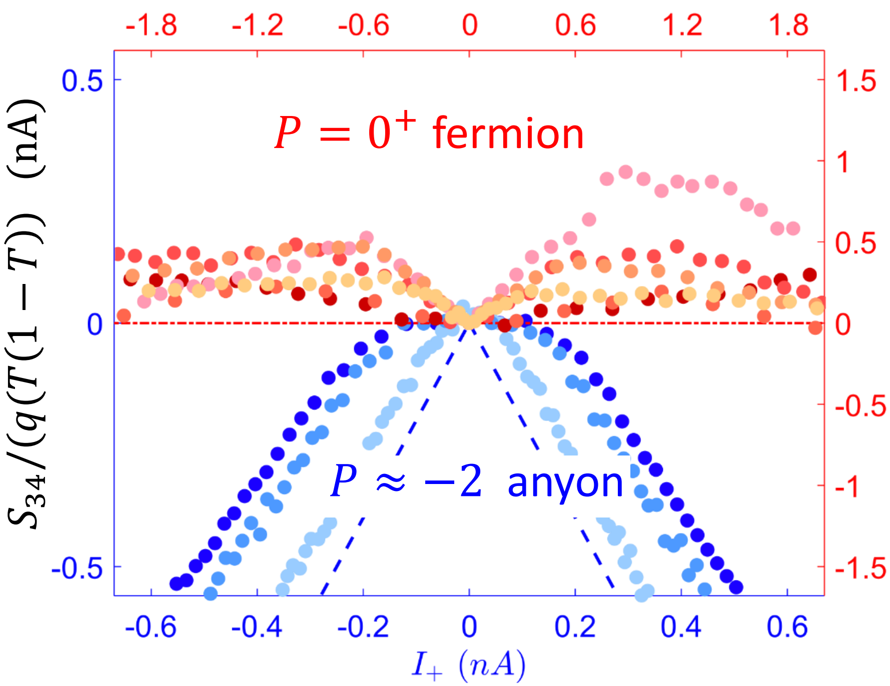
H. Bartolomei et al., Science **368** 173 (2020)

M. Ruelle et al., Phys. Rev. X **13**, 011031 (2023)

See also P. Glidic et al., PRX **13**, 011030 (2023).

Conclusion course 2

- Two-particle interferometry

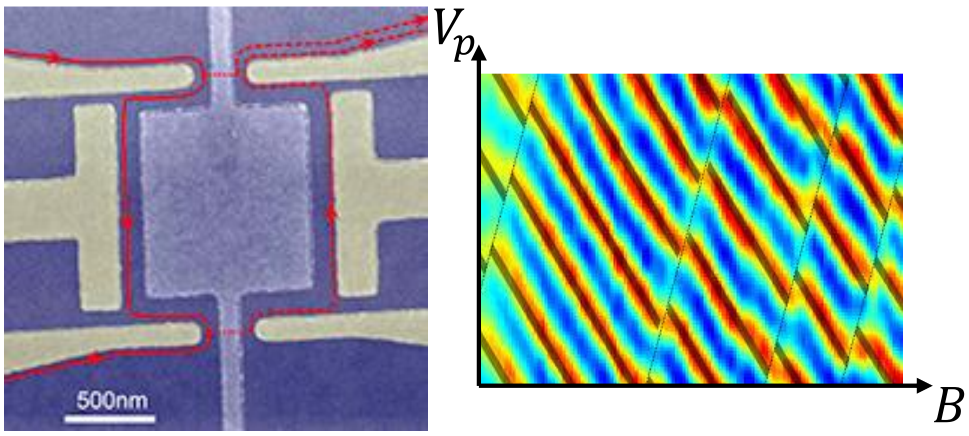


H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

P. Glidic et al., Phys. Rev. X **13**, 011030 (2023).

J.-Y.M. Lee et al., Nature **617**, 277–281 (2023).

- Single particle interferometry



Fabry-Perot interferometer

J. Nakamura, S. Liang, G.C. Gardner, M.J. Manfra, Nature Physics **16** 931 (2020).

Mach-Zehnder interferometer

H.K. Kundu, S. Biswas, N. Ofek, V. Umansky, and M. Heiblum, Nature physics **19**, 515 (2023).

Outlook:

Noise and dynamics : High frequency noise, trigerred anyon emission…

Other filling factors: 2/5, 2/3, 5/2….